Solutions to RSPL/3 (DS2)

1. (*b*), as $5005 = 5 \times 7 \times 11 \times 13 \Rightarrow c = 11$

2. (*a*), as Mode = 3 Median – 2Mean

$\Rightarrow \quad q = 3r - 2p \Rightarrow r = \dfrac{q+2p}{3}$

3. (*b*), as the smallest two digit composite number =10

The largest two digit prime numer = 97

$\therefore \quad LCM\,(10, 97) = 970$

4. (*b*), as if pair of linear equations is consistent, then $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

One equation is $x - 3y - 7 = 0$

Other equation is $3x - y = 7$

5. (*c*), as

$$\sin A = \frac{\sqrt{3}}{2} \Rightarrow A = 60°$$

$$\cos B = \frac{1}{2} \Rightarrow B = 60°$$

$$\tan(A - B) = \tan(60° - 60°) = \tan 0° = 0$$

6. (*d*), as

$$\frac{\cos A - \sin A}{\cos A + \sin A} = \frac{1 - \tan A}{1 + \tan A}$$

[On dividing numerator and denominator by cos *A*]

$$= \frac{1-\sqrt{2}}{1+\sqrt{2}} = \frac{(1-\sqrt{2})^2}{1-2}$$

$$= \frac{1+2-2\sqrt{2}}{-1} = 2\sqrt{2} - 3$$

7. (*c*), as

$$p^2 + q^2 = (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$$

$$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta$$

$$= 1 + 1 = 2$$

8. (*b*), as let coordinates of the other end point be $A(\alpha, \beta)$.

Centre is mid-point of diameter *AB*.

$$\therefore \quad \left(\frac{\alpha+2}{2}, \frac{\beta+0}{2}\right) = (-1, 3)$$

$$\Rightarrow \quad \frac{\alpha+2}{2} = -1, \frac{\beta}{2} = 3$$

$$\Rightarrow \quad \alpha = -4, \beta = 6$$

Hence, coordinates of other end point are (–4, 6).

9. (*c*), as point is (–4, 3)

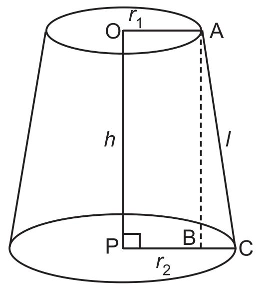
$$\text{Distance from origin} = \sqrt{(-4-0)^2 + (3-0)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5$$

10. (b), as always maximum distance can be equal to the sum of other two distances.

11. $\sqrt{h^2+(r_2-r_1)^2}$, let

$$AB = OP = h$$
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad l^2 = h^2 + (r_2 - r_1)^2$$
$$\Rightarrow \quad l = \sqrt{h^2+(r_2-r_1)^2}$$



12. 3, as given equation is $3x^2 - 2x + 5k = 0$

Product of roots = $\frac{c}{a} = \frac{5k}{3} = 5 \Rightarrow k = 3$

OR

2, as for $y = f(x)$, zeroes are the value of x for which $y = 0$, i.e. point $(x, 0)$ on the x-axis.

$\therefore$ Number of zeroes are 2.

13. FE, as $\Delta ABC \sim \Delta DFE$

We have $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta DFE)} = \frac{BC^2}{FE^2}$

$$\Rightarrow \quad \frac{16}{49} = \frac{BC^2}{FE^2}$$

$$\Rightarrow \quad \frac{BC}{FE} = \frac{4}{7} \Rightarrow BC : FE = 4 : 7 \Rightarrow FE : BC = 7 : 4$$

14. ± 4, as 12, k^2, 20 are in AP.

Then $2k^2 = 12 + 20 \Rightarrow 2k^2 = 32 \Rightarrow k^2 = 16$

$$\Rightarrow \quad k = \pm 4$$

15. $\frac{5}{39}$, as total numbers are 39.

Total possibilities of choosing a number = 39

Favourable possibilities for choosing a number which leaves remainder 2 when divided by 7 are 16, 23, 30, 37, 44, i.e. 5.

$\therefore$ Probability that the number leaves remainder 2 when divided by 7 = $\frac{5}{39}$

16. As numbers a and b are coprime

$\therefore$ HCF$(a, b) = 1$

We have to find least number divisible by a and b, i.e. to find LCM.

$HCF(a, b) \times LCM(a, b)$ = Product of numbers

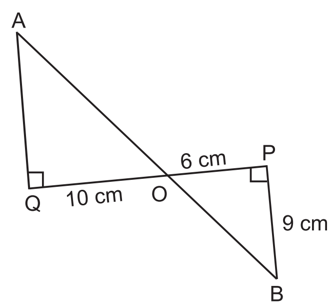
$\Rightarrow LCM(a, b) = a \times b = ab.$

17. $\Delta OAQ \sim \Delta OBP$ (AA similarity)

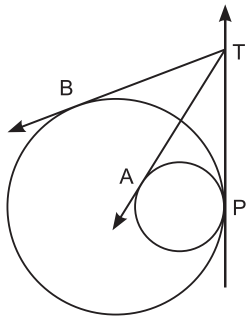
$$\therefore \quad \frac{OA}{OB} = \frac{OQ}{OP} = \frac{AQ}{BP}$$

$$\Rightarrow \quad \frac{10}{6} = \frac{AQ}{9}$$

$$\Rightarrow \quad AQ = \frac{10 \times 9}{6} = 15 \text{ cm}$$



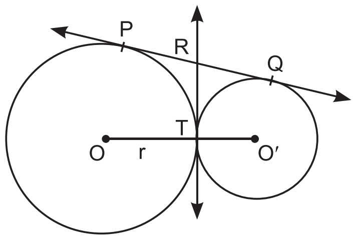
18. We know lengths of the tangents drawn from a point outside the circle to the circle are equal in length.



$\therefore \quad TB = TP$ and $TP = TA$

$\Rightarrow \quad TB = 5$ cm $\Rightarrow TA = 5$ cm

OR



We know lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$\therefore \quad PR = RT = RQ \Rightarrow PR = RQ$

$\Rightarrow$ R is mid-point of PQ.

19. a, b, c are in $AP \Rightarrow 2b = a + c$ ...(*i*)

If $b + c, c + a, a + b$ are in AP.

$\Rightarrow \quad 2(c + a) = b + c + a + b = 4b$ [From (*i*)]

$\Rightarrow \quad c + a = 2b$, which is true from (*i*).

Hence, $b + c, c + a, a + b$ are also in AP.

20. Consider $ax^2 + 2abx = 0$

$\Rightarrow \quad ax(x + 2b) = 0 \Rightarrow ax = 0$ or $x + 2b = 0$

$\Rightarrow \quad x = 0$ or $x = -2b$

21. Given $a_n = 8 - 5n$

$$a_1 = 8 - 5 \times 1 = 3, a_2 = 8 - 10 = -2, \ a_3 = 8 - 15 = -7$$

$\therefore$ AP is 3, – 2, – 7, ...

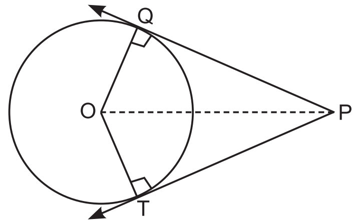
Here, $a = 3, d = -5$

$$\therefore \quad S_7 = \frac{7}{2}[2 \times 3 + (7 - 1)(-5)]$$

$$= \frac{7}{2}(6 - 30) = \frac{7}{2} \times (-24)$$

$$= 7 \times (-12) = -84$$

22. (*i*) PQ and PT are the tangents drawn from point P to the circle with centre O.



In Δs OPQ and OPT

$OQ = OT$ (radii)

OP is common

$\angle OQP = \angle OTP = 90°$ (Tangent is perpendicular to radius at the point of contact.)

$\therefore \quad \Delta OPQ \cong \Delta OPT$

$\therefore \quad PQ = PT$ (CPCT)

(*ii*) $\angle QOT + \angle OQP + \angle QPT + \angle OTP = 360°$

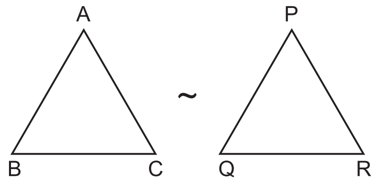
$\angle QOT + 90° + \angle QPT + 90° = 360°$

$\angle QOT + \angle QPT = 180°$

Hence supplementary.

23. Let $\Delta ABC \sim \Delta PQR$

$\Rightarrow \quad \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR} = k$ (say) ...(*i*)



$\Rightarrow \quad AB = kPQ, BC = kQR, AC = kPR$

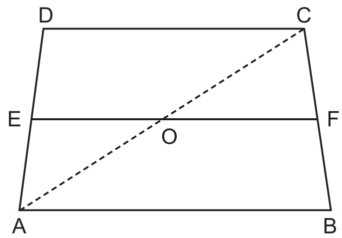
$\Rightarrow \quad AB + BC + AC = k(PQ + QR + PR)$

$\Rightarrow \quad \dfrac{AB+BC+AC}{PQ+QR+PR} = k \Rightarrow \dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = k$

$\therefore \quad \dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR} = \dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR}$

OR

Given: In trapezium $ABCD$, $AB \,||\, DC$ and $EF \,||\, AB$.



To prove: $\dfrac{AE}{ED} = \dfrac{BF}{FC}$

Construction: Join A and C meeting EF at O.

Proof: In ΔABC, $OF \,||\, AB$

$\Rightarrow \quad \dfrac{AO}{OC} = \dfrac{BF}{FC}$ ...(*i*) (BPT Theorem)

Also $\quad EF \,||\, AB$ and $AB \,||\, DC \Rightarrow EF \,||\, DC$

In ΔADC, $\quad OE \,||\, DC$

$\therefore \quad \dfrac{AO}{OC} = \dfrac{AE}{ED}$ ...(*ii*) (BPT Theorem)

$\therefore \quad \dfrac{AE}{ED} = \dfrac{BF}{FC}$ [From (*i*) and (*ii*)]

Hence, any line parallel to parallel sides of a trapezium divides the non parallel sides proportionally.

24.

$$\text{LHS} = \cos 2A = \cos(2 \times 30°) = \cos 60° = \frac{1}{2}$$

$$\text{RHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\tan^2 30°}{1+\tan^2 30°} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{2}{4} = \frac{1}{2}$$

Hence LHS = RHS

25. x red balls + 12 black balls,

3 red balls are added

$\therefore$ Total balls $(x + 3)$ red balls + 12 black balls

Probability of red balls = $\frac{x+3}{x+3+12} = \frac{1}{3}$

$\Rightarrow$ $3x + 9 = x + 15 \Rightarrow 2x = 6 \Rightarrow x = 3$

Number of red balls in the bag initially was 3.

OR

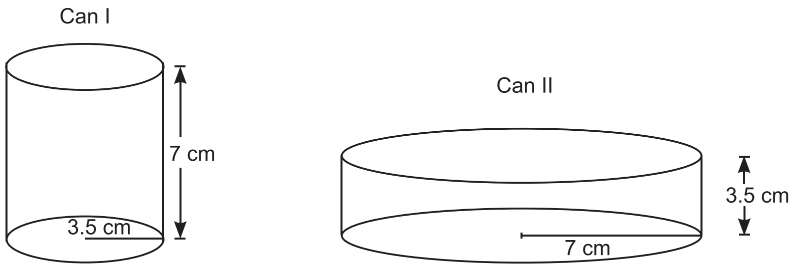
A coin is tossed three times.

$\therefore$ Total outcomes are {*HHH*, *HHT*, *HTH*, *THH*, *HTT*, *THT*, *TTH*, *TTT*}

Favourable cases for head and tail alternately are {*HTH*, *THT*}, i.e. 2

$\therefore$ Probability $= \frac{2}{8} = \frac{1}{4}$

26.



(*i*) For curved surface area

Can I: $2\pi(3.5)(7) = 2 \times \frac{22}{7} \times 3.5 \times 7 = 154 \text{ cm}^2$

Can II: $2\pi(7)(3.5) = 2 \times \frac{22}{7} \times 7 \times 3.5 = 154 \text{ cm}^2$

Same curved surface area.

(*ii*) For volume:

Can I: $\pi(3.5)^2(7) = \frac{22}{7} \times (3.5)^2 \times 7 = 269.5 \text{ cm}^3$

Can II: $\pi(7)^2(3.5) = \frac{22}{7} \times 49 \times 3.5 = 539 \text{ cm}^3$

Can II, by $(539 - 269.5) \text{ cm}^3 = 269.5 \text{ cm}^3$

27. Let, if possible $5-\sqrt{3}$ is a rational number.

Let $5-\sqrt{3} = r$, r is a rational number.

$$\Rightarrow \quad 5 - r = \sqrt{3} \quad ...(i)$$

LHS, we notice $5-r$ is a rational number as difference of two rational numbers is a rational number and RHS, $\sqrt{3}$ is an irrational number.

$\therefore$ From (i), we get a contradiction as rational number is not equal to an irrational number.

Hence, our supposition is wrong. Hence, $5-\sqrt{3}$ is an irrational number.

28. Let a be the first term and d the common difference of a given AP.

$$\begin{aligned} \text{RHS} &= 3(S_8 - S_4) \\ &= 3\left[\frac{8}{2}\{2a+(8-1)d\} - \frac{4}{2}\{2a+(4-1)d\}\right] \\ &= 3(8a + 28d - 4a - 6d) \\ &= 3(4a + 22d) = 6(2a + 11d) \\ &= \frac{12}{2}[2a + (12-1)d] = S_{12} = \text{LHS} \end{aligned}$$

29. Given equations are $\frac{x}{a}+\frac{y}{b} = a + b \Rightarrow bx + ay = ab(a+b)$ ...(i)

and $\frac{x}{a^2}+\frac{y}{b^2} = 2 \Rightarrow b^2x + a^2y = 2a^2b^2$ ...(ii)

We have $bx + ay - ab(a+b) = 0$

$b^2x + a^2y - 2a^2b^2 = 0$

$$\Rightarrow \frac{x}{\begin{vmatrix} a & -ab(a+b) \\ a^2 & -2a^2b^2 \end{vmatrix}} = \frac{y}{\begin{vmatrix} -ab(a+b) & b \\ -2a^2b^2 & b^2 \end{vmatrix}} = \frac{1}{\begin{vmatrix} b & a \\ b^2 & a^2 \end{vmatrix}}$$

$$\Rightarrow \frac{x}{-2a^3b^2 + a^4b + a^3b^2} = \frac{y}{-a^2b^3 - ab^4 + 2a^2b^3} = \frac{1}{a^2b - ab^2}$$

$$\Rightarrow \frac{x}{a^4b - a^3b^2} = \frac{y}{a^2b^3 - ab^4} = \frac{1}{a^2b - ab^2}$$

$$\Rightarrow \frac{x}{a^3b(a-b)} = \frac{y}{ab^3(a-b)} = \frac{1}{ab(a-b)}$$

$$\Rightarrow x = \frac{a^3b(a-b)}{ab(a-b)} = a^2 \text{ and } y = \frac{ab^3(a-b)}{ab(c-b)} = b^2$$

OR

Let man finishes work in x days and boy finishes work in y days.

$$\therefore \quad \frac{2}{x}+\frac{5}{y} = \frac{1}{4} \Rightarrow \frac{6}{x}+\frac{15}{y} = \frac{3}{4} \quad ...(i)$$

$$\text{and} \quad \frac{3}{x}+\frac{6}{y} = \frac{1}{3} \Rightarrow \frac{6}{x}+\frac{12}{y} = \frac{2}{3} \quad ...(ii)$$

Subtracting (ii) from (i), we get

$$\frac{15}{y}-\frac{12}{y} = \frac{3}{4}-\frac{2}{3} \Rightarrow \frac{3}{y} = \frac{1}{12} \Rightarrow y = 36$$

From (i)

$$\frac{2}{x} + \frac{5}{36} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{1}{4} - \frac{5}{36} = \frac{4}{36} = \frac{1}{9}$$

$$\Rightarrow \quad x = 18$$

1 man can finish the work in 18 days.

30. α and β are the zeroes of the polynomial,

$$p(x) = ax^2 + bx + c, a \neq 0$$

$$\therefore \quad \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a} \quad \ldots(i)$$

Consider

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta}$$

$$= \frac{\left(-\frac{b}{a}\right)^3 - 3 \cdot \frac{c}{a}\left(-\frac{b}{a}\right)}{\frac{c}{a}}$$

$$= \frac{\frac{-b^3}{a^3} + \frac{3bc}{a^2}}{\frac{c}{a}}$$

$$= \frac{-b^3 + 3abc}{a^3} \times \frac{a}{c} = \frac{-b^3 + 3abc}{a^2 c}$$

OR

Let $r(x)$ is subtracted

$$\therefore \quad p(x) - r(x) = (4x^2 + 3x - 2) \times \text{quotient}$$

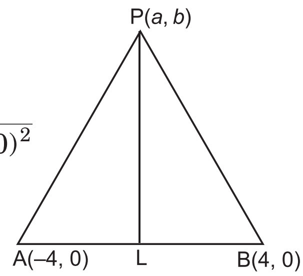
$$\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 8x - 12 \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & - \quad - \quad + \\ \hline
 & 8x^3 + 2x^2 + 8x - 12 \\
 & 8x^3 + 6x^2 - 4x \\
 & - \quad - \quad + \\ \hline
 & -4x^2 + 12x - 12 \\
 & -4x^2 - 3x + 2 \\
 & + \quad + \quad - \\ \hline
 & 15x - 14 \\ \hline
\end{array}$$

$$\therefore \quad p(x) - (15x - 14) = (4x^2 + 3x - 2)(2x^2 + 2x - 1)$$

$\therefore$ $15x - 14$ must be subtracted.

31. (*i*) Given coordinates of point of spot light are $P(a, b)$.

As triangle PAB is equilateral.



$$\therefore \quad PA = AB = PB$$

$$\sqrt{(a+4)^2+(b-0)^2} = \sqrt{(4+4)^2+0^2} = \sqrt{(a-4)^2+(b-0)^2}$$

$$\Rightarrow \quad a^2 + 8a + 16 + b^2 = 64 = a^2 - 8a + 16 + b^2$$

$$\Rightarrow \quad a^2 + 8a + 16 + b^2 = a^2 - 8a + 16 + b^2$$

$$\Rightarrow \quad 16a = 0 \Rightarrow a = 0$$

Also, $a^2 + 8a + 16 + b^2 = 64 \Rightarrow b^2 = 64 - 16 = 48$

$$b = \pm\sqrt{48} = \pm 4\sqrt{3}$$

$\therefore$ Coordinates of P are $(0, \pm 4\sqrt{3})$

(*ii*) As PAB is an equilateral triangle and if PL is distance of P from AB then L is the mid-point of AB.

$\therefore$ coordinates of L are $\left(\frac{-4+4}{2}, \frac{0+0}{2}\right)$, i.e. $(0, 0)$

Distance of P from AB $= PL$

$$= \sqrt{(0-0)^2+(0-4\sqrt{3})^2} = 4\sqrt{3} \text{ units}$$

32. Consider $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

$$\Rightarrow \quad \sin\theta = (\sqrt{2}-1)\cos\theta$$

$$\Rightarrow \quad \sin\theta = \frac{(\sqrt{2}-1)(\sqrt{2}+1)}{\sqrt{2}+1}\cos\theta$$

$$= \frac{2-1}{\sqrt{2}+1}\cos\theta$$

$$\Rightarrow \quad (\sqrt{2}+1)\sin\theta = \cos\theta$$

$$\Rightarrow \quad \sqrt{2}\sin\theta + \sin\theta = \cos\theta$$

$$\Rightarrow \quad \cos\theta - \sin\theta = \sqrt{2}\sin\theta$$

Hence proved

OR

Consider $\frac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40°} + 2\,\text{cosec}^2\, 58° - 2\cot 58° \cdot \tan 32°$
$- 4\tan 13° \cdot \tan 37° \cdot \tan 45° \cdot \tan 53° \cdot \tan 77°$

$$= \frac{\cos^2 20° + \cos^2(90° - 20°)}{\sec^2 50° - \cot^2(90° - 50°)} + 2\,\text{cosec}^2\, 58° - 2\cot 58° \cdot \tan(90° - 58°)$$
$$-4\tan 13° \cdot \tan 37° \cdot 1 \cdot \tan(90° - 37°) \cdot \tan(90° - 13°)$$

$$= \frac{\cos^2 20° + \sin^2 20°}{\sec^2 50° - \tan^2 50°} + 2\,\text{cosec}^2\, 58° - 2\cot 58° \cdot \cot 58° - 4\tan 13° \cdot \tan 37° \cdot \cot 37° \cdot \cot 13°$$

$$= \frac{1}{1} + 2[\text{cosec}^2\, 58° - \cot^2 58°] - 4[(\tan 13° \cdot \cot 13°)(\tan 37° \cdot \cot 37°)]$$

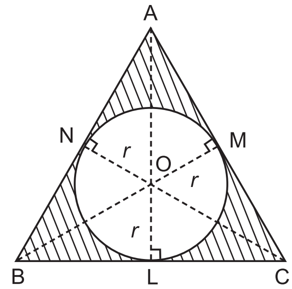
$$= 1 + 2 \times 1 - 4(1)(1) = 1 + 2 - 4 = -1$$

33. $$\text{ar}(\Delta ABC) = \text{ar}(\Delta OAB) + \text{ar}(\Delta OBC) + \text{ar}(\Delta OCA)$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4} \cdot (12)^2 = \frac{1}{2} \cdot r \cdot 12 + \frac{1}{2} \cdot r \cdot 12 + \frac{1}{2} \cdot r \cdot 12$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4} \times 144 = 18r$$

$$\Rightarrow \quad r = \frac{\sqrt{3} \times 36}{18} = 2\sqrt{3} \text{ cm}$$



Hence, radius of the inscribed circle is $2\sqrt{3}$ cm.

$$\therefore \quad \text{Area of circle} = \pi(2\sqrt{3})^2 = 12\pi \text{ cm}^2$$

$$\therefore \quad \text{Area of shaded part} = \frac{\sqrt{3}}{4}(12)^2 - 12\pi$$

$$= 12(3\sqrt{3} - \pi) \text{ cm}^2$$

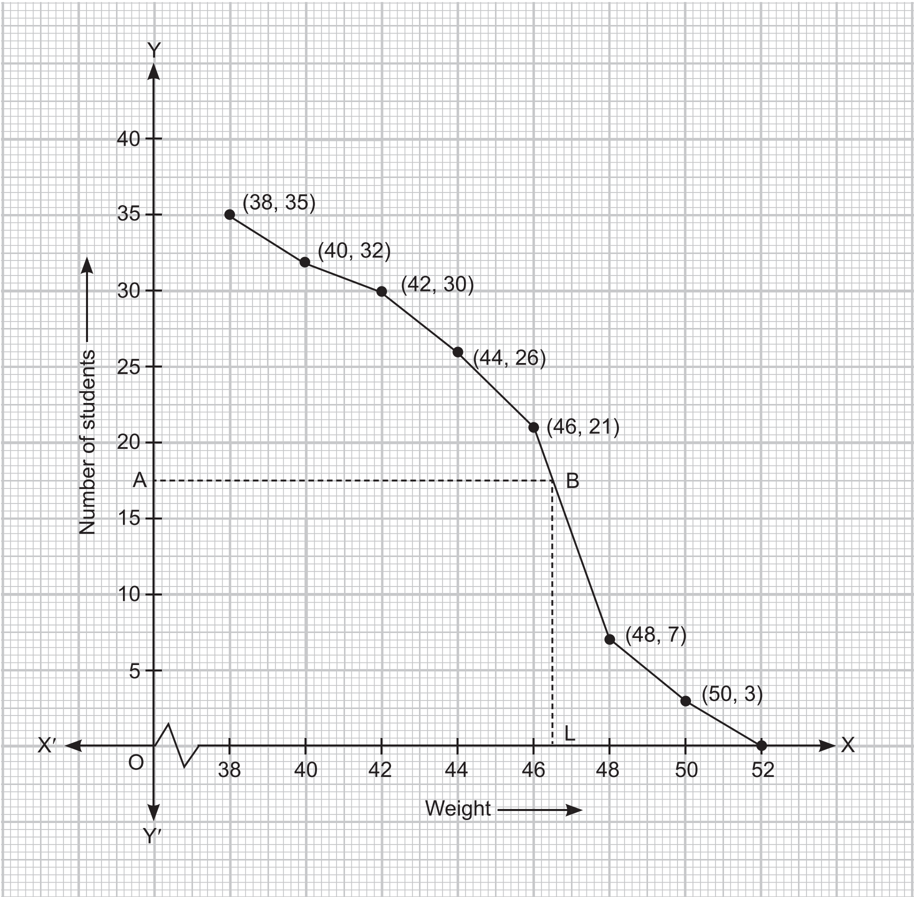
34. Given

Weight (in kg)	**Number of students**	**CI**	f_i
less than 38	0	38 – 40	3
less than 40	3	40 – 42	2
less than 42	5	42 – 44	4
less than 44	9	44 – 46	5
less than 46	14	46 – 48	14
less than 48	28	48 – 50	4
less than 50	32	50 – 52	3
less than 52	35		

Table for more than ogive

Weight (in kg)	**Number of students**	**Points**
more than 38	35	(38, 35)
more than 40	32	(40, 32)
more than 42	30	(42, 30)
more than 44	26	(44, 26)
more than 46	21	(46, 21)
more than 48	7	(48, 7)
more than 50	3	(50, 3)
more than 52	0	(52, 0)

(*i*) More than ogive



(*ii*) For median $\frac{N}{2} = \frac{35}{2} = 17.5$

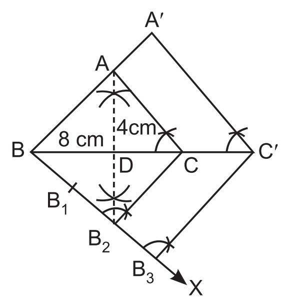
We draw a line AB from $A(0, 17.5)$ parallel to x-axis meeting the curve at B and BL parallel to the y-axis meeting x-axis at L.

Coordinates of L are (46.5, 0) its median is 46.5

(*iii*) This represents 50% class has weight less than 46.5 kg and 50% class has weight more than 46.5 kg.

Note: If a student has written only 50% class has weight less than 45.5 kg or 50% class has weight more than 46.5 kg, then give marks.

35. Steps of construction:



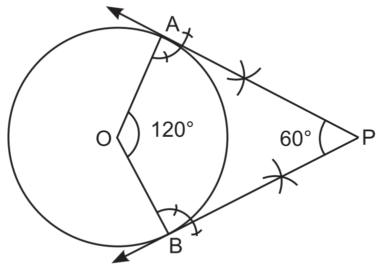
1. Base BC = 8 cm is drawn.
2. Perpendicular bisector of BC is drawn meeting BC at D.
3. AD = 4 cm is cut.
4. AB and AC are joined. ΔABC is an isosceles triangle.
5. Below BC, $\angle CBX$ is drawn.
6. On BX, B_1, B_2, B_3 are drawn such that $BB_1 = B_1B_2 = B_2B_3$
7. B_2 and C are joined

8. B_3C' is drawn parallel to B_2C meeting BC produced at C'.
9. $C'A'$ is drawn parallel to CA meeting BA produced at A'.

   Then $\Delta A'BC'$ is required triangle.

OR

Steps of construction:



1. A circle of radius 5 cm is drawn with centre O.
2. Radii OA and OB are drawn at an angle of $(180° - 60°) = 120°$.
3. At A, $\angle OAP = 90°$ is drawn.
4. At B, $\angle OBP = 90°$ is drawn meeting AP at P.

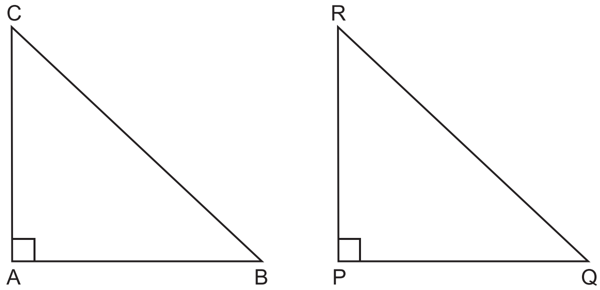
   Then AP and BP are required tangents at 60°.

36. Statement: In a triangle, if square on one side is equal to the sum of the sqaures on the other two sides, then the angle opposite to the first side is a right angle.

Given: A ΔABC, such that $BC^2 = AB^2 + AC^2$

To prove: $\angle A$ is a right angle.

Construction: construct a triangle PQR such that $\angle P = 90°$, $PR = AC$ and $PQ = AB$.



Proof: ΔPQR is right angled at P

$\therefore$	$QR^2 = PR^2 + PQ^2$	(Pythagoras Theorem)
$\Rightarrow$	$QR^2 = AC^2 + AB^2$	(From construction)...(*i*)
Also,	$BC^2 = AC^2 + AB^2$	(Given)...(*ii*)
$\Rightarrow$	$QR^2 = BC^2$	[From (*i*), (*ii*)]
$\Rightarrow$	$QR = BC$	

Consider triangle ABC and PQR

	$PR = AC$	(Construction)
	$PQ = AB$	(Construction)
	$QR = BC$	(Proved above)
$\therefore$	$\Delta ABC \cong \Delta PQR$	(SSS)
$\Rightarrow$	$\angle A = \angle P$	
But	$\angle P = 90°$	
$\Rightarrow$	$\angle A = 90°$	

Hence ΔABC is right angled at A.

37. Let numbers be x(larger) and $(16 - x)$ smaller.

According to given condition

$$2x^2 = (x - 16)^2 + 164$$

$$\Rightarrow \quad 2x^2 = x^2 - 32x + 256 + 164$$

$$\Rightarrow \quad x^2 + 32x - 420 = 0$$

$$\Rightarrow \quad (x + 42)(x - 10) = 0$$

$$\Rightarrow \quad x + 42 = 0, x - 10 = 0$$

$$\Rightarrow \quad x = -42 \text{ (rejected)}; x = 10$$

$\therefore$ Larger part = 10, smaller part = 16 – 10 = 6

OR

Let duration of tour be x days

Total expenses = ₹ 360

$\therefore$ Expense per day = ₹ $\frac{360}{x}$

Tour is increased by 4 days

$\therefore$ Expenses per day = ₹ $\frac{360}{x+4}$

According to given condition

$$\frac{360}{x} - \frac{360}{x+4} = 3$$

$$\Rightarrow \quad \frac{360x + 1440 - 360x}{x(x+4)} = 3$$

$$\Rightarrow \quad x^2 + 4x = 480 \Rightarrow x^2 + 4x - 480 = 0$$

$$\Rightarrow \quad (x + 24)(x - 20) = 0$$

$$\Rightarrow \quad x + 24 = 0 \text{ or } x - 20 = 0$$

$$\Rightarrow \quad x = -24 \text{ (rejected) or } x = 20$$

$\therefore$ Tour is for 20 days.

38. Diameter of base of cone and hemisphere = 4 cm

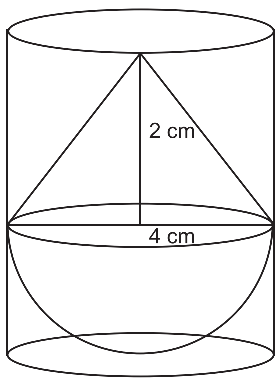
$\therefore$ Radius of base = 2 cm

Height of the cone = 2 cm

Volume of solid toy = volume of hemisphere + volume of cone

$$= \frac{2}{3}\pi(2)^3 + \frac{1}{3}\pi(2)^2 \cdot 2$$

$$= \frac{3}{3}\pi \times 8 = 8\pi \text{ cm}^3$$



If right circular cylinder circumscribes the toy. Then

Radius of base of cylinder = 2 cm

Height of cylinder = (2 + 2) cm = 4 cm

$\therefore$ Volume of cylinder $= \pi(2)^2 \cdot 4 = 16\pi \text{ cm}^3$

Volume of excess space $= (16\pi - 8\pi) \text{ cm}^3 = 8\pi \text{ cm}^3$

OR

Let x smaller spherical balls be made.

Radius of solid sphere = 3 cm

Radius of smaller spherical ball = $\frac{0.6}{2}$ cm = 0.3 cm

(*i*) Number of small balls × volume of a small spherical ball = volume of given solid sphere

$$\therefore \quad x \times \frac{4}{3}\pi(0.3)^3 = \frac{4}{3}\pi(3)^3$$

$$\Rightarrow \quad x = \frac{3 \times 3 \times 3}{0.3 \times 0.3 \times 0.3} = 1000$$

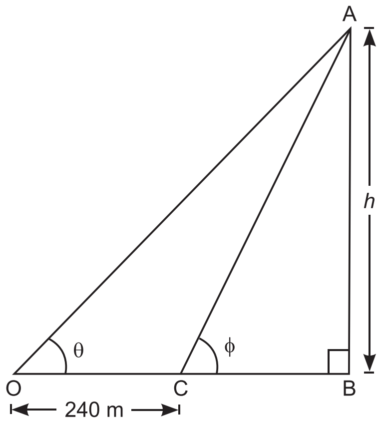
1000 small spherical balls are made.

(*ii*) Surface area of solid sphere = $4\pi(3)^2 = 36\,\pi$ cm^2

Total surface area of 1000 small spherical balls = $1000 \times 4\pi\,(0.3)^2 = 360\pi$ cm^2

$\therefore$ Difference of surface area = $(360\pi - 36\pi)\,\text{cm}^2 = 324\pi\,\text{cm}^2$

39. Let height of lighthouse be h m



In right-angled triangle *OBA*

$$\frac{h}{OB} = \tan\theta = \frac{5}{12}$$

$$\Rightarrow \quad OB = \frac{12h}{5} \qquad \ldots(i)$$

In right-angled triangle *CBA*.

$$\frac{h}{BC} = \tan\phi = \frac{3}{4}$$

$$\Rightarrow \quad BC = \frac{4h}{3} \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get

$$\frac{12h}{5} - \frac{4h}{3} = 240$$

$$\Rightarrow \quad 36\,h - 20\,h = 240 \times 15$$

$$\Rightarrow \quad 16\,h = 240 \times 15$$

$$\Rightarrow \quad h = \frac{240 \times 15}{16}$$

$$= 15 \times 15 = 225 \text{ m}$$

$\therefore$ Height of lighthouse is 225 m.

40. We make classes continuous

CI	*f*	*cf*
0.5 – 4.5	2	2
4.5 – 8.5	5	7
8.5 – 12.5	8	15
12.5 – 16.5	9	24
16.5 – 20.5	12	36
20.5 – 24.5	14	50
24.5 – 28.5	14	64 ← Median class
28.5 – 32.5	15	79
32.5 – 36.5	11	90
36.5 – 40.5	13	103
	103	

$$\frac{N}{2} = \frac{103}{2} = 51.5$$

Median class is 24.5 – 28.5

$$\therefore \quad \text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h$$

$$= 24.5 + \frac{51.5 - 50}{14} \times 4$$

$$= 24.5 + \frac{6}{14} = 24.5 + 0.43 = 24.93$$