Solution

Class 10 - Mathematics

Sample Paper 3

Section A

1. (c)
   $0 \leqslant r \ < \ 3$

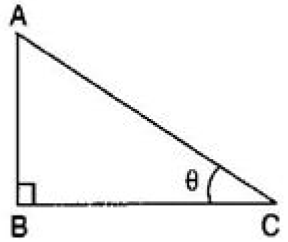
   Explanation:
   Since a is a positive integer, therefore, r = 0, 1, 2 only.
   So, that a = 3q, 3q + 1, 3q + 2.
2. (b)
   6

   Explanation:
   Let n be a positive integer,
   then three consecutive positive integers are $(n+1)(n+2)(n+3)$ =
   $n(n+1)(n+2)+3(n+1)(n+2)$
   Here, the first term is divisible by 6 and the second term is also divisible by 6
   Because it contains a factor 3 and one of the two consecutive integers $(n+1)$ or $(n+2)$ is even and thus is divisible by 2
   Therefore, the sum of multiple of 6 is also a multiple of 6.
3. (c)
   $\frac{x_i-a}{h}$

   Explanation:
   In the formula, $\overline{x} \ = \ a+h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$ for finding the mean,
   $u_i = \frac{x_i-a}{h}$. where xi = mid values of class intervals
   a = assumed mean values
   x_i - a = deviation of each mid - value from the assumed mean value
   h = class size
4. (a)
   2p + q, 2p – q

   Explanation:
   Given: $x^2-4px+4p^2-q^2=0$
   $\Rightarrow(x-2p)^2-q^2=0$
   Using a^2 - b^2 = (a+b) (a-b),
   $\Rightarrow(x-2p+q)(x-2p-q)=0$
   $\Rightarrow x-2p+q=0$ and $x-2p-q=0$
   $\Rightarrow x=2p-q$ and $x=2p+q$
5. (a)
   60°

   Explanation:



Let the height of the pole AB = $\sqrt{3}x$ meters and length of the string AC = 2x meters.
An angle of elevation be θ
$\therefore \sin\theta = \frac{\mathrm{AB}}{\mathrm{AC}}$
$\Rightarrow \sin\theta = \frac{\sqrt{3}x}{2x}$
$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$
$\Rightarrow \sin\theta = \sin 60^\circ$
$\Rightarrow \theta = 60^\circ$

6. (b)
   $\cos^2\theta$

   Explanation:
   Given: $\frac{\cos\theta\cos(90^\circ-\theta)}{\cot(90^\circ-\theta)}$
   = $\frac{\cos\theta\sin\theta}{\tan\theta}$
   = $\frac{\cos\theta\sin\theta}{\sin\theta} \times \cos\theta$
   = $\cos^2\theta$
7. (c)
   2

   Explanation:
   Given: $\cot\mathrm{A} + \frac{1}{\cot\mathrm{A}} = 2$
   Squaring both sides, we get
   $\Rightarrow \cot^2\mathrm{A} + \frac{1}{\cot^2\mathrm{A}} + 2 \times \cot\mathrm{A} \times \frac{1}{\cot\mathrm{A}} = 4$
   $\Rightarrow \cot^2\mathrm{A} + \frac{1}{\cot^2\mathrm{A}} = 2$
8. (c)
   $\frac{1}{2}$

   Explanation:
   Number of possible outcomes = {2, 3, 5} = 3
   Number of Total outcomes = 6
   $\therefore$ Probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$
9. (b)
   1

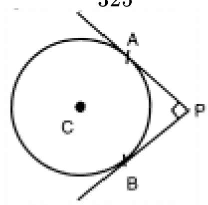
   Explanation:
   Let A$(a, 0)$, B$(0, b)$ and C (1, 1).
   Since points A, B and C are collinear, then ar $(\Delta\mathrm{ABC}) = 0$
   $\Rightarrow \frac{1}{2}|a(b-1) + 0(1-0) + 1(0-b)| = 0$
   $\Rightarrow |ab - a - b| = 0$
   $\Rightarrow ab - a - b = 0 \Rightarrow a + b = ab$
   $\Rightarrow \frac{1}{a} + \frac{1}{b} = 1$
10. (b)
    4

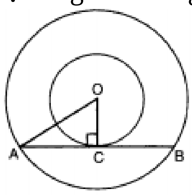
Explanation:
Let the coordinates of midpoint O$(5, p)$ is equidistance from the points A$(3p, 4)$ and B$(-2, 4)$.(because O is the mid-point of AB)
$\therefore 5 = \frac{3p-2}{2} \Rightarrow 3p - 2 = 10$
$\Rightarrow 3p = 12 \Rightarrow p = 4$
Also $p = \frac{4+4}{2} \Rightarrow p = 4$

11. $\frac{4}{3}\pi(R^3 - r^3)$
12. a = 2

OR

3

13. 1
14. $a_n = S_n - S_{n-1}$
15. circumference
16. $\frac{619}{325} = \frac{619}{5^2 \times 13}$
Here, the denominator is $5^2 \times 13$ is not of the form $2^m \times 5^n$, where m and n are non-negative integers.
Hence, $\frac{619}{325}$ will have a non-terminating repeating decimal expansion.
17. 

Construction: Join AC and BC
Now, $AC \perp AP \text{ and } CB \perp BP$
$\angle APB = 90^0$
Therefore, CAPB will be a square
CA = AP = PB = BC = 4 cm
$\therefore$ Length of tangent = 4 cm.
18. 

Now, In $rt.\triangle\ OCA$
$AO^2 = OC^2 + AC^2$
$\Rightarrow AC^2 = 5^2 - 3^2$
$\Rightarrow AC = 4$
We know, $OC \perp AB$
$\therefore AC = BC$
Hence, $AC = 2(4) = 8cm$
19. Given AP = $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}$ ...
1^{st} term = $\frac{3}{2}$
Common difference = $\frac{1}{2} - \frac{3}{2}$ = -1

OR

0.6, 1.7, 2.8, 3.9...
First term = a= 0.6
Common difference (d) = Second term -First term
= Third term - Second term and so on
Therefore, Common difference (d) = 1.7−0.6=1.1
20. Here $a = k, b = 4, c = 1$
$D = b^2 - 4ac$

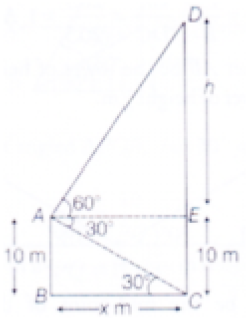
= 4^2 - 4 $\times$ k $\times$ 1
$= 16 - 4k.$
For equal roots, D = 0
$\Rightarrow$ 16 - 4k = 0
$\Rightarrow$ k = 4.

Section B

21. Given $\angle BOC = 45^o$
$\angle AOC = 180^o - 45^o = 135^o$
Area of circle = πr^2
Area of region X $= \frac{\theta}{360^\circ}\pi r^2$
$= \frac{135^\circ}{360^\circ}\pi r^2$
=$\frac{3}{8}\pi$r^2
$\therefore$Probability that the spinner will land in the region X $= \frac{\text{Area of region } x}{\text{Area of circle}}$ = $= \frac{\frac{3}{8}\pi r^2}{\pi r^2}$ = $\frac{3}{8}$.
22. We know that the radius and tangent are perpendicular at their point of contact
$\because \angle OBP = \angle OAP = 90^\circ$
Now, In quadrilateral AOBP
$\angle APB + \angle AOB + \angle OBP + \angle OAP = 360^\circ$
$\Rightarrow \angle APB + \angle AOB + 90^\circ + 90^\circ = 360^\circ$
$\Rightarrow \angle APB + \angle AOB = 180^\circ$
Since, the sum of the opposite angles of the quadrilateral is 180°
Hence, AOBP is a cyclic quadrilateral.
23. In $\triangle$PQR, CA $\parallel$ PR
$\therefore \quad \frac{PC}{CQ} = \frac{RA}{AQ}$ (By BPT)
or, $\frac{PC}{15} = \frac{20}{12}$
$\therefore \quad PC = \frac{15\times 20}{12}$ = 25 cm
In $\triangle$PQR, CB | | QR
$\therefore \quad \frac{PC}{CQ} = \frac{PB}{BR}$(By BPT)
or, $\frac{25}{15} = \frac{15}{BR}$
$\therefore \quad BR = \frac{15\times 15}{25}$ = 9 cm.

OR

In$\triangle ABC, AB \| DE.$
$\therefore \quad \frac{CD}{DA} = \frac{CE}{EB}$ ...(i) [by Thales' theorem]
In $\triangle CDB, BD \| EF$
$\therefore \quad \frac{CF}{FD} = \frac{CE}{EB}$ ...(ii) [by Thales' theorem]
From (i) and (ii) we get
$\frac{CD}{DA} = \frac{CF}{FD}$
$\Rightarrow \quad \frac{DA}{DC} = \frac{FD}{CF}$ [taking reciprocals]
$\Rightarrow \quad \frac{DA}{DC} + 1 = \frac{FD}{CF} + 1$
$\Rightarrow \frac{DA+DC}{DC} = \frac{FD+CF}{CF}$
$\Rightarrow \quad \frac{AC}{DC} = \frac{DC}{CF}$
$\Rightarrow \quad DC^2 = CF \times AC$
24. Let a man is standing on the deck of a ship at point A such hat AB = 10 cm and let CD be the hill.
Then, $\angle EAD = 60^o$
and $\angle CAE = \angle BCA = 30^o$ [alternate angles]
Let BC = x cm = AE and DE= h m



In right-angled $\triangle ABC$,

$\tan 30^o = \frac{P}{B} = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{x}$

$\Rightarrow$ x = 10 $\sqrt{3}$

In right-angled $\triangle AED$,

$\tan 60^o = \frac{P}{B} = \frac{DE}{EA} = \frac{h}{x}$

$\Rightarrow \sqrt{3} = \frac{h}{10\sqrt{3}}$

$\Rightarrow h = 10 \times 3$

$\Rightarrow h = 30\text{m}$

Height of Hill = 10 + 30 = 40 cm

Distance of the hill from the ship is $10\sqrt{3}$ m and height of the hill is 40 m.

25. We have,

a(x^2 + 1) - x(a^2 + 1) = 0

$\Rightarrow$ ax^2 + a - xa^2 - x = 0

$\Rightarrow$ ax^2 - xa^2 + a - x = 0

$\Rightarrow$ ax(x - a) - 1(x - a) = 0

$\Rightarrow$ (x - a)(ax - 1) = 0

$\Rightarrow$ either x - a = 0 or, ax - 1 = 0

$\Rightarrow$ x = a or $x = \frac{1}{a}$

Hence, the roots of given quadratic equation are a and $\frac{1}{a}$

OR

Let the total number of camels be x

According to the question,

$\frac{x}{4} + 2\sqrt{x} + 15 = x$

or, $3x - 8\sqrt{x} - 60 = 0$

let $\sqrt{x} = y$,then

$3y^2 - 8y - 60 = 0$

or, $3y^2 - 18y + 10y - 60 = 0$

or,$3y(y - 6) + 10(y - 6) = 0$

or, $(3y + 10)(y - 6) = 0$

or, $y = 6$ or $y = -\frac{10}{3}$ (not possible)

So, $y = 6$ or, $y^2 = 36$

$x = y^2 = 36$

Hence the number of camels = 36.

26. We have, Inner diameter of the glass, d = 5 cm, Height of the glass = 10 cm

i. The apparent capacity of the glass = Volume of cylinder

= $\pi r^2 h$

= $3.14 \times \left(\frac{5}{2}\right)^2 \times 10$

$= 3.14 \times \frac{25}{4} \times 10 = 196.25 \text{ cm}^3$

ii. The actual capacity of glass = Apparent capacity of glass - Volume of hemispherical part of the glass

The volume of hemispherical part = $\frac{2}{3}\pi r^2 h$ = $\frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3 = 32.71 \text{ cm}^3$

Actual capacity of glass = 196.25 - 32.71 = 163.54 cm^3

Section C

27. Let $\frac{1}{2+\sqrt{3}}$ be a rational number.

A rational number can be written in the form of $\frac{p}{q}$ where p,q are integers.

$\frac{1}{2+\sqrt{3}} = \frac{p}{q}$

$\Rightarrow \sqrt{3} = \frac{q-2p}{p}$

p, q are integers then $\frac{q-2p}{p}$ is a rational number.

Then $\sqrt{3}$ is also a rational number.

But this contradicts the fact as $\sqrt{3}$ is an irrational number.

So, our supposition is false.

Therefore, $\frac{1}{2+\sqrt{3}}$ is an irrational number.

OR

(1). Let us consider: rational × irrational

Let R be any rational number

Case I : R=0 then $R = 0 = \frac{0}{5} = \frac{0}{100} = \frac{0}{-25}$ (So 0 is a rational number)

$0 \times \sqrt{2} = 0$

Rational ×irrational=rational

Case II If R ≠ 0 then

for example:

$3 \times \sqrt{2} = \sqrt{2} + \sqrt{2} + \sqrt{2}$ (irrational)

$\frac{2}{3} \times \sqrt{3} = \frac{\sqrt{3}+\sqrt{3}}{3}$ (irrational)

$\frac{15}{13} \times \sqrt{13}\ irrational$

Hence the product of a rational number and an irrational number can be a rational number or an irrational number.

(2) Irrational x Irrational

The product of two irrational numbers, in some cases, will be irrational. However, it is possible that product of two irrational numbers may be rational.

For example

$\sqrt{5} \times \sqrt{5} = 5$(rational)

$\sqrt{3} \times \sqrt{2} = \sqrt{6}\ (irrational)$

$\sqrt{2} \times \sqrt{2} = 2$ (rational)

$\sqrt{5} \times \sqrt{2} = \sqrt{10}$ (irrational)

28. Let A (1, -2) and B (-3,4) be the given points.

Let the points of trisection be P and Q. Then, AP = PQ = QB = X(say)

PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = λ : 2λ = 1 : 2 and AQ : QB = 2λ : λ = 2 : 1

So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2: 1. Thus, the coordinates of P and Q are

$P\left(\frac{1\times-3+2\times1}{1+2}, \frac{1\times4+2\times-2}{1+2}\right) = P\left(\frac{-1}{3}, 0\right)$

$Q\left(\frac{2\times-3+1\times1}{2+1}, \frac{2\times4+1\times(-2)}{2+1}\right) = Q\left(\frac{-5}{3}, 2\right)$

Hence, the two points of trisection are (-1/3, 0) and (-5/3, 2)

29. Let father's age be x years and the sum of the ages of his two children be y years.

Then,

$x = 2y$

$\Rightarrow x - 2y = 0$ ....(i)

20 years hence,

Father's age = $(x + 20) years$

Sum of the ages of two children $= y + 20 + 20 = (y + 40) years$

Then, we have

$x + 20 = y + 40$

$\Rightarrow x - y = 20$....(ii)

Multiplying (ii) by 2, we get

$2x - 2y = 40$ ...(iii)

Subtracting (i) from (iii), we have

x = 40

Thus, the age of father is 40 years.

OR

According to question the given system of equations are

$\frac{x}{10} + \frac{y}{5} - 1 = 0$

$\Rightarrow \frac{x+2y-10}{10} = 0$

$\Rightarrow$ x + 2y = 10 ....(i)

And, $\frac{x}{8} + \frac{y}{6} = 15$

$\Rightarrow \frac{3x+4y}{24} = 15$

$\Rightarrow$ 3x + 4y = 360 ....(ii)

Multiplying equation (i) by 3, we get

3x + 6y = 30 ....(iii)

Subtracting equation (iii) from (ii), we get

3x + 4y - 3x - 6y = 360 - 30

-2y = 330

$\Rightarrow$ y = -165

Substitute the value of y = -165 in (i), we get

x + 2(-165) = 10

$\Rightarrow$ x - 330 = 10

$\Rightarrow$ x = 340

Now it is given that $y = \lambda x + 5$

$\Rightarrow -165 = \lambda \times 340 + 5$

$\Rightarrow 340\lambda = -170$

$\Rightarrow \lambda = \frac{-170}{340} = -\frac{1}{2}$

Hence, x = 340, y = -165 and $\lambda = -\frac{1}{2}$.

30. Let p(x) = x^2 - 2x - (7p + 3)

Since -1 is a zero of p(x). Therefore,

p(-1) = 0

$(-1)^2$ - 2(-1) - (7p + 3)=0

1 + 2 - 7p - 3 = 0

3 - 7p - 3 = 0

7p = 0

p = 0

Thus, p(x) = x^2 - 2x - 3

For finding zeros of p(x), we put,

p(x) = 0

x^2 - 2x - 3 = 0

x^2 - 3x - x - 3 = 0

x(x-3) + 1(x-3) = 0

(x-3)(x+1) = 0

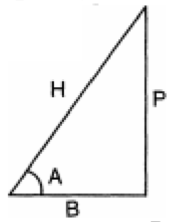
Put x - 3 = 0 and x + 1 = 0, we get,

Thus, x = 3, -1

Thus, the other zero is 3.

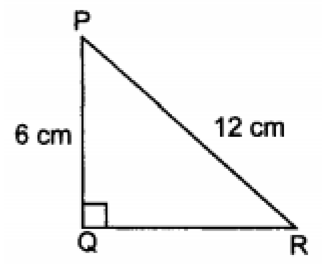
31. $S_n = \frac{3n^2}{2} + \frac{13n}{2}$
$S_n = \frac{3n^2+13n}{2}$
$a_n = S_n - S_{n-1}$
or, $a_{25} = S_{25} - S_{24}$
$= \frac{3(25)^2+13(25)}{2} - \frac{3(24)^2+13(24)}{2}$
$= \frac{1}{2}[3(25^2 - 24^2) + 13(25 - 24)]$
$= \frac{1}{2}[3(625 - 576) + 13(1)]$
$= \frac{1}{2}(3 \times 49 + 13)$
$= \frac{1}{2}(147 + 13)$
$= \frac{1}{2}(160)$
= 80

32. If two expressions are equal for all the values of same parameter or parameters , then the statement of equality between the two expressions is called an identity.



Let, $\tan A = \frac{P}{B}$, $\sec A = \frac{H}{B}$
$H^2 = P^2 + B^2$
LHS $= 1 + \tan^2 A$
$= 1 + \left(\frac{P}{B}\right)^2 = 1 + \frac{p^2}{B^2}$
$= \frac{B^2+P^2}{B^2} = \frac{H^2}{B^2}$
$= \left(\frac{H}{B}\right)^2$
$= \sec^2 A$
= R.H.S
Hence Proved.

OR



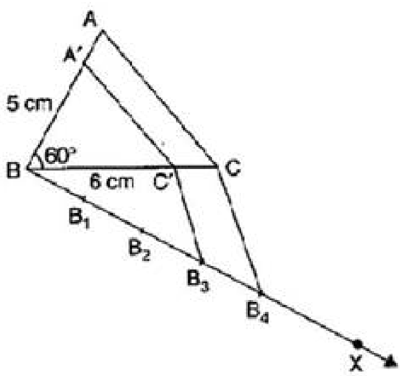
In $\triangle$PQR
$\sin(\angle \mathrm{PRQ}) = \frac{\mathrm{PQ}}{\mathrm{PR}}$
$\Rightarrow \sin(\angle \mathrm{PRQ}) = \frac{6}{12} = \frac{1}{2}$
Also, $\sin 30° = \frac{1}{2}$
$\Rightarrow \angle PRQ = 30^\circ$
In right $\triangle$PQR, $\angle Q + \angle R + \angle P = 180^\circ$
$90^\circ + 30^\circ + \angle P = 180^\circ$
$\Rightarrow \angle P = 60^\circ$
$\therefore \angle \mathbf{QPR} = 60^\circ, \angle \mathbf{PRQ} = 30^\circ$

33.
Radius of a pond $= \frac{17.5}{2} = 8.75$
Area of a pond $= \pi(8.75)^2\ sq.m$

Radius of a circle including path = 8.75 + 2 = 10.75 m
According to question,
Area of the path = Area of a circle including path - Area of a pond
$= \pi(10.75)^2 - \pi(8.75)^2$
$= \pi\left[(10.75)^2 - (8.75)^2\right]$
$= \frac{22}{7}[(10.75 + 8.75)(10.75 - 8.75)]$
$= \frac{22}{7}[19.5 \times 2]$
$= \frac{22}{7} \times 39$
$= \frac{858}{7} sq.m$
= 122.5 sq.m
Cost of constructing the path = 25×122.5= Rs.3062.50

34. According to question we are given that Cards marked with numbers 5 to 50 are placed in a box and mixed thoroughly. A card is drawn from the box at random. Therefore All possible outcomes are 5, 6, 7, 8 .... 50.
Number of all possible outcomes = 46
    i. Out of the given numbers, the prime numbers less than 10 are 5 and 7.
    Suppose E_1 be the event of getting a prime number less than 10.
    Then, number of favorable outcomes = 2
    Therefore, P(getting a prime number less than 10) = P(E) = $\frac{2}{46} = \frac{1}{23}$
    ii. Out of the given numbers, the perfect squares are 9, 16, 25, 36 and 49.
    Suppose E_2 be the event of getting a perfect square.
    Then, number of favorable outcomes = 5
    Therefore, P(getting a perfect square) = P(E) = $\frac{5}{46}$

Section D

35. 

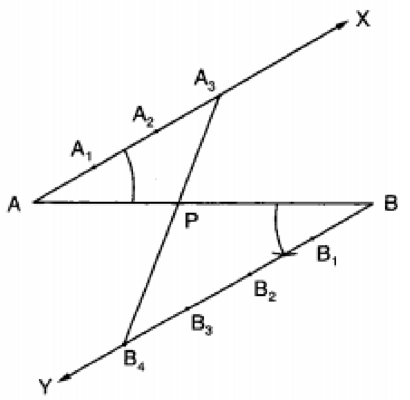
To construct: To construct a triangle ABC with side BC = 6 cm, AB = 5 cm and$\angle ABC = 60^\circ$ then a triangle similar to it whose sides are $\frac{3}{4}$ of the corresponding sides of the first triangle ABC.
Steps of construction:
    i. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^\circ$.
    ii. From any ray BX, making an acute angle with BC on the side opposite to the vertex A.
    iii. Locate 4 points B_1, B_2, B_3 and B_4 on BX such that $BB_1 = B_1 B_2 = B_2 B_3 = B_3 B_4$.
    iv. Join B_4 C and draw a line through the point B_3, draw a line parallel to B_4 C intersecting BC at the point C'.
    v. Draw a line through C' parallel to the line CA to intersect BA at A'.
    Then, A'BC' is the required triangle.
Justification :
$\because B_4C || B_3C'$ [By construction]
$\therefore \frac{BB_3}{BB_4} = \frac{BC'}{BC}$ [By Basic Proportionality Theorem]
But $\frac{BB_3}{BB_4} = \frac{3}{4}$ [By construction]
Therefore, $\frac{BC'}{BC} = \frac{3}{4}$ .... (i)
$\because$ CA || C'A' [By construction]

$\therefore \triangle BC'A \sim \triangle BCA$ [AA similarity]
$\therefore \frac{AB'}{AB} = \frac{A'C'}{AC} = \frac{BC'}{BC} = \frac{3}{4}$ [From eq. (i)]

OR

Steps of construction
STEP I Draw the line segment AB of length 8 cm.
STEP II Draw any ray AX making an acute angle $\angle$BAX with AB.
STEP III Draw a ray BY parallel to AX by making $\angle ABY$ equal to $\angle BAX$.
STEP IV Mark the three point A_1, A_2, A_3 on AX and 4 points B_1, B_2, B_3, B_4 on BY such that

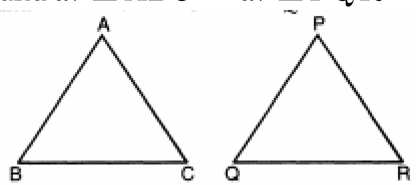
$AA_1 = A_1A_2 = A_2A_3 = BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.



STEP V Join A_3 B_4. Suppose it intersects AB at a point P.

Then, P is the point dividing AB internally in the ratio 3:4.

36. Given:
$\Delta ABC \sim \Delta PQR$
and $ar\,\Delta\,ABC = ar\,\Delta\,PQR$



To prove: $\Delta ABC \cong \Delta PQR$
Proof: $\Delta ABC \sim \Delta PQR$
Also $\text{ar}(\Delta ABC) = \text{ar}(\Delta PQR)$ (given)
or, $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$
Or $\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} = 1$
Or $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1$
Hence we get that
AB=PQ,BC=QR and CA=RP
Hence $\Delta ABC \cong \Delta PQR$

37. Let the length and breadth of the rectangular park be 'L' and 'B' respectively.
Case 1-
If the length and the breadth of the park is decreased by 2 m respectively, its area is decreased by 196 square meter.
Therefore, the area will be = (L $\times$ B) - 196
(L - 2) (B - 2) = L $\times$ B - 196
LB - 2B - 2L +4 = LB -196
- 2B - 2L = - 196 - 4
- 2B - 2L = - 200 ....(1)
Case 2-
If the length of the park is increased by 3 m and breadth is increased by 2 m, then its area is increased by 246 square meter.
Therefore, the area will be = (L $\times$ B) + 246

(L + 3) (B + 2) = L $\times$ B + 246
LB + 3B + 2L +6 = LB + 246
3B + 2L = 246 - 6
3B + 2L = 240 ....(2)
Adding equation (1) and (2), we get, B = 40
Substituting the value B = 40 in equation (2), we get
3B + 2L = 240
3 $\times$ 40 + 2L = 240
120 + 2L = 240
2L = 240 - 120
2L = 120
L = 60
So, the length and breadth of the park is 60 m and 40 m respectively.

OR

λx + y = λ^2 and x + λy = 1
We know that the system of linear equations
$a_1x + b_1y + c_1 = 0$...........(i)
$a_2x + b_2y + c_2 = 0$...........(ii)

i. has no solution,if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
   $\Rightarrow \underset{\text{I}}{\frac{\lambda}{1}} = \underset{\text{II}}{\frac{1}{\lambda}} \neq \underset{\text{III}}{\frac{\lambda^2}{1}}$
   From I and II, we get
   λ^2 = 1
   ⇒ λ = ±1.
   but from II & III; we get
   $\lambda^3 \neq 1$
   $\Rightarrow \lambda \neq 1$
   $\Rightarrow \lambda = -1$
ii. has infinitely many solutions, if
   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
   $\Rightarrow \frac{\lambda}{1} = \frac{1}{\lambda} = \frac{\lambda^2}{1}$
   ⇒ λ^2 = 1 (from I & II part)
   ⇒ λ = ±1 ....(a)
   Also, $\frac{\lambda}{1} = \frac{\lambda^2}{1}$ (from I & III part)
   ⇒ λ^2 = λ
   ⇒ λ^2 - λ = 0
   ⇒ λ(λ - 1) = 0
   ⇒ λ = 0, λ = 1..........(b)
   ∴ Common solution from equations (a) &(b) for which the pair of linear equations has infinitely many solutions is λ = 1 only.
iii. For a unique solution, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
   $\therefore \frac{\lambda}{1} \neq \frac{1}{\lambda}$
   ⇒ λ^2 ≠ 1 or λ ≠ 1, -1
   So, for the unique solution all real value except λ = 1, -1.

38. Diameter of cylindrical pipe = 7 cm
∴ Radius of cylindrical pipe = $\frac{7}{2}$cm = 3.5 cm
Volume of water flows in 1 min = 192.5 litres
Volume of water that flows per hour = (192.50×60)litres
$= (192.50 \times 60 \times 1000)\text{cm}^3$ ....(I)
Let h cm be the length of the column of water that flows in one hour.

Clearly, water column forms a cylinder of radius 3.5 cm and length h cm.

∴ Volume of water that flows in one hour

= πr^2 h

Volume of the cylinder of radius 3.5 cm and length h cm

$= \left[\frac{22}{7} \times (3.5)^2 \times h\right] \text{cm}^3$ ...(ii)

From (i) and (ii), we have

πr^2 h = 192.50 × 60 × 1000

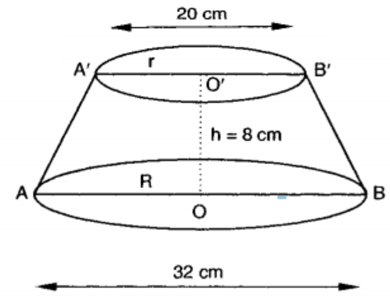
$\frac{22}{7} \times 3.5 \times 3.5 \times h = 192.50 \times 60 \times 1000$

$\Rightarrow \quad h = \frac{192.50 \times 60 \times 1000 \times 7}{22 \times 3.5 \times 3.5}$cm = 300000 cm = 3 km

Hence, the rate of flow of water is 3 km per hour.

OR

Let ABB 'A' be the friction clutch of slant height l cm.



We have,

R = 16 cm, r = 10 cm and h = 8 cm

$\therefore \quad l^2 = h^2 + (R - r)^2$

$= (8)^2 + (16 - 10)^2$

= 64 + $(6)^2$

$\Rightarrow \quad l^2 = 64 + 36 \Rightarrow l = 10\text{cm}$

Bearing surface of the clutch = Lateral surface of the frustum

$\Rightarrow \quad S = \pi(R + r)l$

$= \frac{22}{7} \times (16 + 10) \times 10\text{cm}^2$

$= 817.14\text{cm}^2$

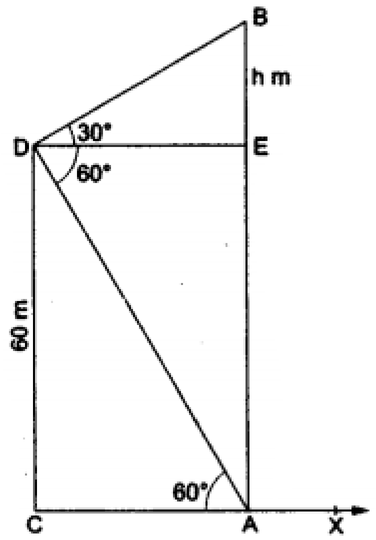
$V = \frac{1}{3}\pi h\left(R^2 + Rr + r^2\right)$

$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 8 \times \left(16^2 + 16 \times 10 + 10^2\right) \text{cm}^3$

$\Rightarrow \quad V = \frac{1}{3} \times \frac{22}{7} \times 8 \times (256 + 160 + 100)\text{cm}^3 = \frac{176}{21} \times 516\text{cm}^3$

= 4324.57 cm^3

39. Let us suppose that AB be the tower and CD be the building. Then, CD = 60m.



Suppose CAX be the horizontal ground.
Draw $DE \perp AB$.
Now, $\angle EDB = 30^\circ$(given)
and $\angle CAD = \angle ADE = 60^\circ$.(given)
Also, it is given that AE = CD = 60m.
Let us suppose that BE = h metres.
From right ΔACD,we have
$\frac{CA}{CD} = \cot 60^\circ = \frac{1}{\sqrt{3}}$
$\Rightarrow \frac{CA}{60m} = \frac{1}{\sqrt{3}} \Rightarrow CA = \frac{60}{\sqrt{3}}$ ....(i)
From right ΔEDB, we have
$\frac{DE}{BE} = \cot 30^\circ = \sqrt{3} \Rightarrow \frac{DE}{h} = \sqrt{3}$
$\Rightarrow DE = h\sqrt{3}\text{m}$ ....(ii)
But, according to question it is given that CA = DE.
∴from (i) and (ii), we get
$\frac{60}{\sqrt{3}} = h\sqrt{3} \Rightarrow 3h = 60 \Rightarrow h = 20$
Hence, the difference between the heights of the tower and the building is 20 metres.
Again from (i), we have$CA = \left(\frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right)$ (by rationalization)
= 20 √3 metres
Hence, the distance between the tower and the building is $20\sqrt{3}$

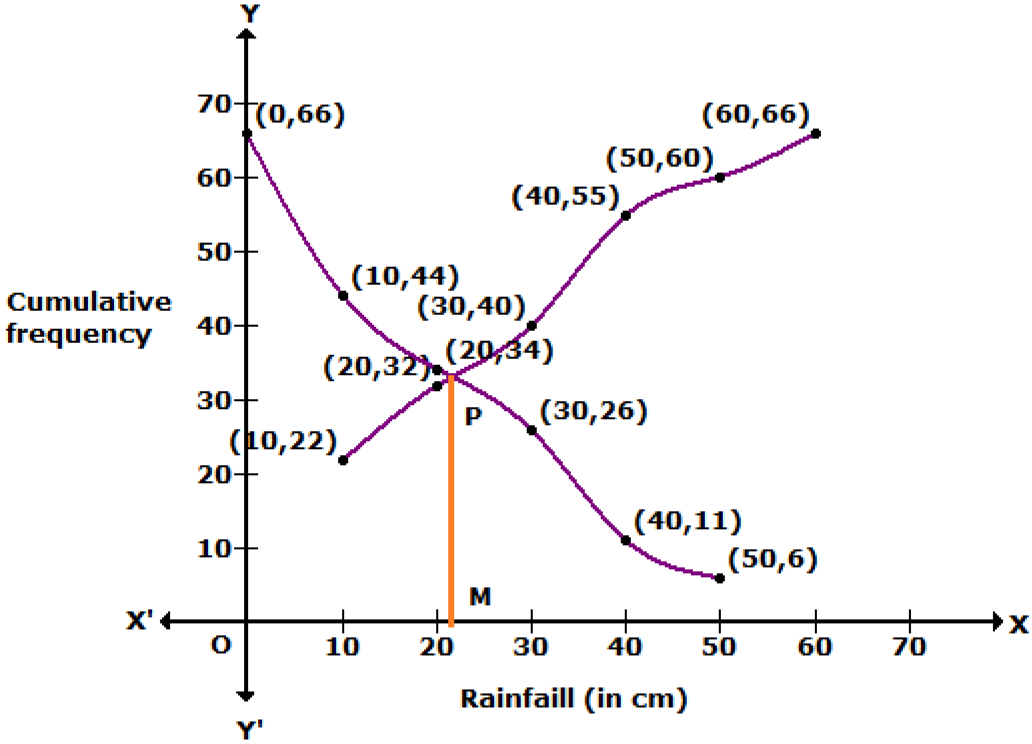
40. More Than Series:

Class Interval	Cumulative Frequency
More than 0	66
More than 10	44
More than 20	34
More than 30	26
More than 40	11
More than 50	6

We plot the points (0, 66), (10, 44), (20, 34), (30, 26), (40, 11), and (50, 6) to get more than ogive.
Less Than Series:

Class Interval	Cumulative Frequency
Less than 10	22
Less than 20	32
Less than 30	40
Less than 40	55
Less than 50	60
Less than 60	66

We plot the points (10, 22), (20, 32), (30, 40), (40, 55), (50, 60) and (60, 66) to get less than ogive.



From the graph, median = 21.25 cm