Solution

Class 10 - Mathematics

Sample Paper 2

Section A

1. (a)
   0.5

   Explanation:
   $9^{x+2} = 240 + 9^x$
   $\Rightarrow 9^x.9^2 = 240 + 9^x$
   $\Rightarrow 9^x(81 - 1) = 240$
   $\Rightarrow 9^x = 3$
   $\Rightarrow 9^x = 9^{\frac{1}{2}}$
   $\Rightarrow x = \frac{1}{2} = 0.5$

2. (b)
   a rational number

   Explanation:
   $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$ = $\left\{\left(\sqrt{a}\right)^2 - \left(\sqrt{b}\right)^2\right\}$
   = $(a - b)$
   Since a and b both are positive rational numbers,
   therefore difference of two positive rational numbers is also rational.

3. (d)
   62

   Explanation:
   $\overline{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$
   = $47.5 + \frac{29}{30} \times 15$
   =47.5+$\frac{29}{2}$
   = 47.5 + 14.5
   = 62

4. (c)
   30 and 35

   Explanation:
   Let one multiple of 5 be x then the next consecutive multiple of will be $(x+5)$ According to question,
   $x(x+5) = 1050$
   $\Rightarrow x^2 + 5x - 1050 = 0$
   $\Rightarrow x^2 + 35x - 30x - 1050 = 0$
   $\Rightarrow x(x+35) - 30(x+35) = 0$
   $\Rightarrow (x-30)(x+35) = 0$
   $\Rightarrow x - 30 = 0$ and $x + 35 = 0$
   $\Rightarrow x = 30$ and $x = -35$
   $x = -35$ is not possible therefore $x = 30$
   Then the other multiple of 5 is $x + 5 = 30 + 5 = 35$
   Then the number are 30 and 35.

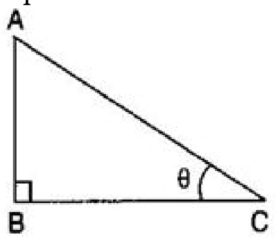
5. (a)
   90°

Explanation:
Given: $\sin\alpha = \frac{1}{\sqrt{2}}$
$\Rightarrow \sin\alpha = \sin 45^\circ$
$\Rightarrow \alpha = 45^\circ$
And $\cos\beta = \frac{1}{\sqrt{2}}$
$\Rightarrow \cos\beta = \cos 45^\circ$
$\Rightarrow \beta = 45^\circ$
$\therefore \alpha + \beta = 45^\circ + 45^\circ = 90^\circ$

6. (d)
$\tan 60^0$

Explanation:
$\frac{2tan\ 30^0}{1-tan^2\ 30^0}$
$= \frac{2\times\frac{1}{\sqrt{3}}}{1-(\frac{1}{\sqrt{3}})^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$
$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = tan\ 60^0$

7. (b)
60°

Explanation:


Given: the height of the pole = AB = 6 m and the length of the shadow = BC = $2\sqrt{3}$ m
$\therefore \tan\theta = \frac{\text{AB}}{\text{BC}}$
$\Rightarrow \tan\theta = \frac{6}{2\sqrt{3}}$
$\Rightarrow \tan\theta = \frac{3}{\sqrt{3}}$
$\Rightarrow \tan\theta = \frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$
$\Rightarrow \tan\theta = \sqrt{3}$
$\Rightarrow \tan\theta = \tan 60^\circ$
$\Rightarrow \theta = 60^\circ$

8. (a)
– 5

Explanation:
Since $x-$coordinate of a point is called abscissa.
Therefore, abscissa is -5.

9. (c)
abscissa

Explanation:
The distance of a point from the y – axis is the x (horizontal) coordinate of the point and is called abscissa.

10. (a)
0.24

Explanation:
Given: P (It will rain on a particular day) = 0.76
∴ P (It will not rain on a particular day) = 1 – P (It will rain particular day)
= 1 – 0.76 = 0.24

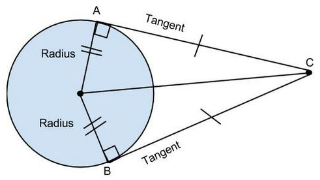
11. 2(l + b)h sq units
12. -3

OR

$\frac{-1}{2}$

13. equilateral
14. $\sqrt{3}$
15. $4\sqrt{2}$
16. Clearly, the given number 23.123456789 is a terminating decimal. So, it is rational.
Therefore, it can be expressed in $\frac{p}{q}$, where q = 10^9 and factors of q are of the form $2^m \times 5^n$, where n and m are positive integers.
17. Given: AB = 5.6 cm, AD = 1.4 cm, AC = 7.2 cm and AE = 1.8 cm
$\therefore \quad \frac{AD}{AB} = \frac{1.4}{5.6} = \frac{1}{4}$ and $\frac{AE}{AC} = \frac{1.8}{7.2} = \frac{1}{4}$
$\Rightarrow \quad \frac{AD}{AB} = \frac{AE}{AC}$
Hence, by the converse of Thales' theorem, $DE \| BC$.
18. Here, $a = p \; and \; d = q$
We know that $a_n = a_1 + (n-1)d$
a_{10} = p + (10 - 1)q = p+ 9q
Hence, 10th term of the AP is p + 9q.

OR

Here, $a = -37, \; d = -33 - (-37) = -33 + 37 = 4 \; and \; n = 12$
Now we know that , $S_n = \frac{n}{2}$ [2a+(n-1)d]
Therefore, $S_{12} = \frac{12}{2}[2 \times (-37) + (12-1)4]$
= $6[-74 + 44]$
= 6 × (-30)
= -180
Therefore, sum of given A.P is -180.
19. 

Consider C is an external point as shown in figure.
According to theorem, from an external point only two tangents can be drawn to a circle.
So, value of k = 2
20. $x^2 + 6x + 9 = 0$.
Put x = -3 in the equation
$\Rightarrow (-3)^2 + 6(-3) + 9$
$\Rightarrow 9 - 18 + 9 = 0$
Hence, it is a solution of the given equation.

Section B

21. H- Heads T- Tails
Total number of outcomes = 8 (HHH, HHT, HTH, THH, HTT, THT, TTH, TTT)
Favourable number of outcomes (HHH, TTT) = 2
Probability (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$
∴ Probability (losing the game) = 1− P(getting success) = 1- $\frac{1}{4}$ = $\frac{3}{4}$

22. We have,

$kx^2 - 2\sqrt{5}x + 4 = 0$

Here, a = k, $b = -2\sqrt{5}$ and c = 4

$\therefore D = b^2 - 4ac$

$= (-2\sqrt{5})^2 - 4 \times k \times 4$

= $20 - 16k$

$\Rightarrow D = 20 - 16k$

The given equation will have real and equal roots, if

$D = 0$

$\Rightarrow 20 - 16k = 0$

$\Rightarrow 16k = 20$

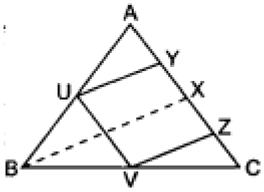
$\Rightarrow k = \frac{20}{16} = \frac{5}{4}$

$\therefore k = \frac{5}{4}$

23. Join BX

In ABX, U is midpoint of AB and Y is mid-point AX (given)

$\therefore$ UY $\|$ BX (using mid-point theorem) .....(i)



In BCX, v is mid-point of BC and z is mid-point of XC

VZ $\|$ BX ..(ii)

from (i) and (ii)

UY $\|$ VZ

In ABC, U is mid-point of AB and V is mid-point of BE.

$\therefore$ UV $\|$ AC

$\Rightarrow$ UV $\|$ YZ Hence proved.

OR

Given: ABC and DBC are two triangles on the same base BC but on the opposite sides of BC, AD intersects BC at O.

construction: Draw AL $\perp$ BC and DM $\perp$ BC

To prove: $\frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{AO}{EO}$

Proof: In $\triangle$ALO and $\triangle$DMO

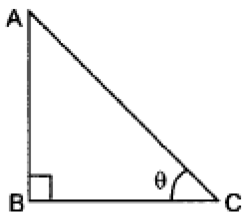
$\angle$ ALO = $\angle$ DMO [Each 90^{o}]

$\angle$ AOL =$\angle$DOM [Vertically opposite angles]

$\therefore \triangle$ ALO $\sim$ $\triangle$DMO [By AA similarity]

$\Rightarrow \frac{AL}{DM} = \frac{AO}{DO}$

$$\therefore \frac{ar(\triangle ABC)}{ar(\triangle DBC)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times BC \times DM} = \frac{AL}{DM} = \frac{AO}{DO}$$

24. 

Let AB is the tower, $AB = 15m, BC = 15m$

In right $\triangle$ABC, $\tan\theta = \frac{AB}{BC}$

$\Rightarrow \tan\theta = \frac{15}{15}$

$\Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^\circ$

25. Given: PQ and PR are tangents to a circle with centre O and $\angle$QPR = 50°.

To find: $\angle$QSR

$\angle$QOR + $\angle QPR = 180^\circ$

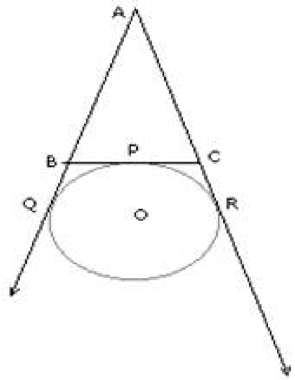
$\Rightarrow \angle QOR + 50^\circ = 180^\circ$

$\Rightarrow \angle QOR = 130^\circ$

$\Rightarrow \angle$QSR = $\frac{1}{2}$ $\angle$QOR

$\Rightarrow \angle$QSR = $\frac{1}{2} \times 130^\circ = 65^\circ$

OR



We know that the two tangents drawn to a circle from an external point are equal.

$\therefore$ AQ = AR, BP = BQ, CP = CR

$\therefore$ Perimeter of $\triangle$ABC = AB + BC + AC

= AB + BP + PC + AC

= AB + BQ + CR + AC [$\because$ BP = BQ, PC = CQ]

= AQ + AR = 2AQ = 2AR = [$\because$ AQ = AR]

= AQ = AR = $\frac{1}{2}$[Perimeter of $\triangle$ABC]

26. For cone, Radius of the base (r)

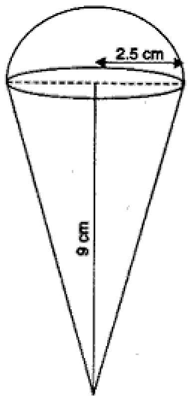
$= 2.5\text{cm} = \frac{5}{2}\text{cm}$

Height (h) = 9 cm

$\therefore$ Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times 9$

$= \frac{825}{14}\text{cm}^3$



For hemisphere,

Radius (r) = $2.5\text{cm} = \frac{5}{2}\text{cm}$

$\therefore$ Volume = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{5}{2} = \frac{1375}{42}\text{cm}^3$

i. The volume of the ice-cream without hemispherical end = Volume of the cone

$= \frac{825}{14}\text{cm}^3$

ii. Volume of the ice-cream with hemispherical end = Volume of the cone + Volume of the hemisphere

$= \frac{825}{14} + \frac{1375}{42} = \frac{2475+1375}{42}$

$= \frac{3850}{42} = \frac{275}{3} = 91\frac{2}{3}\text{cm}^3$

Section C

27. We can prove $7\sqrt{5}$ irrational by contradiction.
Let us suppose that $7\sqrt{5}$ is rational.
It means we have some co-prime integers a and b ($b \neq 0$)
such that
$7\sqrt{5} = \frac{a}{b}$
$\Rightarrow \sqrt{5} = \frac{a}{7b}$ ....(1)
R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.
It is not possible which means our supposition is wrong.
Therefore, $7\sqrt{5}$ cannot be rational.
Hence, it is irrational.

OR

Let take that $3 + 2\sqrt{5}$ is a rational number.
So we can write this number as
$3 + 2\sqrt{5} = \frac{a}{b}$
Here a and b are two co-prime numbers and b is not equal to 0.
Subtract 3 both sides we get,
$2\sqrt{5} = \frac{a}{b} - 3$
$2\sqrt{5} = \frac{a-3b}{b}$
Now divide by 2 we get
$\sqrt{5} = \frac{a-3b}{2b}$
Here a and b are an integer so $\frac{a-3b}{2b}$ is a rational number so $\sqrt{5}$ should be a rational number but $\sqrt{5}$ is an irrational number so it contradicts the fact.
Hence the result is $3 + 2\sqrt{5}$ is an irrational number
Now its square will again contain an irrational number.
Hence the given number is an irrational number.

28. Let the 1st term of AP be a and common difference be d.
Now, three middle terms of this AP are a_{10}, a_{11} and a_{12}
from question,we have,
$a_{10} + a_{11} + a_{12} = 129$
$\Rightarrow (a + 9d) + (a + 10d) + (a + 11d) = 129$
$\Rightarrow 3a + 30d = 129$
$\Rightarrow a + 10d = 43 \Rightarrow a = 43 - 10d$ ....(i)
Also, last three terms are a_{19}, a_{20} and a_{21}
$\therefore a_{19} + a_{20} + a_{21} = 129$
$\Rightarrow (a + 18d) + (a + 19d) + (a + 20d) = 237$
$\Rightarrow 3a + 57d = 237$
$\Rightarrow a + 19d = 79 \Rightarrow 43 - 10d + 19d = 79$ [using eq. (i)]
$\Rightarrow 9d = 36 \Rightarrow d = 4$
When d = 4, equation (i) becomes
$a = 43 - 10(4) = 3$
$\therefore$ AP is 3, 7, 11, 15, ...

29. $4x + 6y = 3xy$
$\Rightarrow \frac{4x+6y}{xy} = 3$
$\frac{4}{y} + \frac{6}{x} = 3$ ....(i)
$\frac{8x+9y}{xy} = 5$
$\frac{8}{y} + \frac{9}{x} = 4$ ....(ii)
Putting $\frac{1}{x}$=u and $\frac{1}{y}$= v ,the given equations

$4u + 6v = 3$ ....(iii)
$8u + 9v = 5$......(iv)
Multiplying (iii) by 9 and (iv) by 6, we get
$36v + 54u = 27$........(v)
$48v + 54u = 30$.......(vi)
Subtracting (iii) from (iv),we get
$12v = 3$
$v = \frac{3}{12} = \frac{1}{4}$
Putting v = $\frac{1}{4}$ in (iii), we get
$4 \times \frac{1}{4} + 6u = 3$
$1 + 6u = 3$
$6u = 3 - 1 = 2$
$u = \frac{2}{6} = \frac{1}{3}$
Now, $u = \frac{1}{x} \Rightarrow \frac{1}{x} = \frac{1}{3} \Rightarrow x = 3$
and,$v = \frac{1}{y} \Rightarrow \frac{1}{y} = \frac{1}{4} \Rightarrow y = 4$
Hence, the solution is $\mathrm{x} = 3, \mathrm{y} = 4$

OR

We have to solve the following systems of equations by using the method of substitution:
$\frac{2x}{a} + \frac{y}{b} = 2$
$\frac{x}{a} - \frac{y}{b} = 4$
The given system of equations are:
$\frac{2x}{a} + \frac{y}{b} = 2$......(i)
$\frac{x}{a} - \frac{y}{b} = 4$.......(ii)
From equation (i), we get
$\frac{y}{b} = 2 - \frac{2x}{a}$
$\Rightarrow y = b\left(2 - \frac{2x}{a}\right)$
Substituting $y = b\left(2 - \frac{2x}{a}\right)$in equation (ii), we get
$\frac{x}{a} - \frac{b}{b}\left(2 - \frac{2x}{a}\right) = 4$
$\Rightarrow \quad \frac{x}{a} - 2 + \frac{2x}{a} = 4$
$\Rightarrow \quad \frac{3x}{a} = 6$
$\Rightarrow$ 3x = 6a
$\Rightarrow$ x = 2a
Putting x = 2a in equation (i), we get
$4 + \frac{y}{b} = 2$
$\Rightarrow \frac{y}{b} = -2$
$\Rightarrow y = -2b$
Hence, the solution of the given system of equations is x = 2a and y = -2b.

30. It is given f(x)= $6x^3 + 11x^2 - 39x - 65$ and g(x) = $x^2 - 1 + x$.
So now on long division of f(x) by g(x)

$x^2 + x - 1 \,\big)\, 6x^3 + 11x^2 - 39x - 65 \,\big(\, 6x + 5$
$6x^3 + 6x^2 - 6x$
$- \quad - \quad +$
$5x^2 - 33x - 65$
$5x^2 + 5x - 5$
$- \quad - \quad +$
$-38x - 60$

Hence quotient q(x)=6x+5 and remainder r(x)=-38x-60
Also, g(x) q(x) + r(x) = $(x^2 + x - 1)(6x + 5) + (-38x - 60)$
= $6x^3 + 6x^2 - 6x + 5x^2 + 5x - 5 - 38x - 60$

i.e. f(x) = g(x) q(x) + r(x) = $6x^3 + 11x^2 - 39x - 65$

or, Dividend = Quotient × Divisor + Remainder

31. If points are collinear, then one point divides the other two in the same ratio.

Let point (m, 6) divides the join of (3, 5) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ in the ratio k: 1.

Then, (m, 6) = $\left(\frac{\frac{k}{2}+3}{k+1}, \frac{15k}{k+1}\right)$

$\Rightarrow$ m = $\frac{\frac{k}{2}+3}{k+1}$ ...(i)

and 6 = $\frac{\frac{15}{2}k+5}{k+1}$ ...(ii)

From (ii), we get 6k + 6 = $\frac{15k}{2} + 5$

$\Rightarrow 6k - \frac{15k}{2}$ = - 1

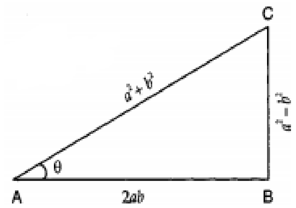
$\Rightarrow -\frac{3}{2}k$ = - 1

$\Rightarrow \quad k = \frac{2}{3}$

Substituting, k = $\frac{2}{3}$ in (i), we get

m = $\frac{\frac{1}{2}\times\frac{2}{3}+3}{\frac{2}{3}+1} = \frac{\frac{1}{3}+3}{\frac{2}{3}+1} = \frac{\frac{10}{3}}{\frac{5}{3}}$ = 2

Hence, for m = 2 points are collinear.

32. 

We have,

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a^2-b^2}{a^2+b^2}$

So, Let us draw a right triangle ABC in which $\angle B$ is right angle, we have

Perpendicular = BC = $a^2 - b^2$ Hypotenuse = AC = $a^2 + b^2$ and, $\angle BAC = \theta$

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad \left(a^2+b^2\right)^2 = AB^2 + \left(a^2-b^2\right)^2$

$\Rightarrow \quad AB^2 = \left(a^2+b^2\right)^2 - \left(a^2-b^2\right)^2$

$\Rightarrow \quad AB^2 = \left(a^4+b^4+2a^2b^2\right) - \left(a^4+b^4-2a^2b^2\right)$

$\Rightarrow \quad AB^2 = 4a^2b^2 = (2ab)^2$

$\Rightarrow \quad AB = 2ab$

Now, Let$\angle BAC = \theta$

We have

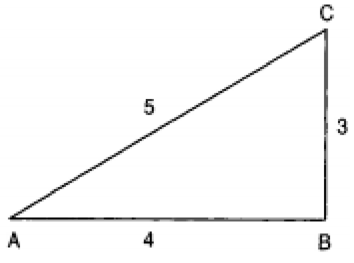
Base = AB = 2ab, Perpendicular $= \text{BC} = a^2 - b^2$, Hypotenuse = AC = $a^2 + b^2$

Therefore, $\quad \cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{2ab}{a^2+b^2}, \tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a^2-b^2}{2ab}$

$\quad cosec\ \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{a^2+b^2}{a^2-b^2}, \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{a^2+b^2}{2ab}$

and, $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{2ab}{a^2-b^2}$

OR



According to the question,

$\sin A=\frac{\text{Perpendicular}}{\text{Hypotenuse}}=\frac{3}{5}$

Draw a right triangle right angled at B such that

Perpendicular $=BC=3$ and, Hypotenuse $=AC=5$

Using Pythagoras theorem,

$\Rightarrow AC^2=AB^2+BC^2$

$\Rightarrow \quad 5^2=AB^2+3^2$

$\Rightarrow \quad AB^2=25-9=16$

$\Rightarrow \quad AB=\sqrt{16}=4$

Consider the trigonometric ratios of $\angle C$, we have

Base $=BC=3$, Perpendicular $=AB=4$ and, Hypotenuse $=AC=5$.

$\therefore \quad \sin C=\frac{\text{Perpendicular}}{\text{Hypotenuse}}=\frac{4}{5}, \quad \cos C=\frac{\text{Base}}{\text{Hypotenuse}}=\frac{3}{5}$

$\tan C=\frac{\text{Perpendicular}}{\text{Base}}=\frac{4}{3}$, $\operatorname{cosec} C=\frac{\text{Hypotenuse}}{\text{Perpendicular}}=\frac{5}{4}$

$\sec C=\frac{\text{Hypotenuse}}{\text{Base}}=\frac{5}{3}$ and, $\cot C=\frac{\text{Base}}{\text{Perpendicular}}=\frac{3}{4}$

33. We have, Rate of fencing = Rs 24 per metre and, Total cost of fencing = Rs 5280

$\therefore$ Length of the fence $=\frac{\text{Total cost}}{\text{Rate}}=\frac{5280}{24}$ = 220 metre

$\Rightarrow$ Circumference of the field = 220 metre

$\Rightarrow 2\pi r$ = 220, where r is the radius of the field

$\Rightarrow \quad 2\times\frac{22}{7}\times r=220$

$\Rightarrow \quad r=\frac{220\times 7}{22\times 2}=35$

Area of the field = $\pi r^2=\frac{22}{7}\times 35\times 35\text{m}^2=22\times 5\times 35\text{m}^2$

It is given that the field is ploughed at the rate of Rs 0.50 per m^2

Cost of ploughing the field = Rs $(22\times 5\times 35\times 0.50)$= Rs 1925

34. The given series is an inclusive series. Making it an exclusive series by subtracting 0.5 from lower limit and adding 0.5 in upper limit, we get

Class Interval(Exclusive)	**Frequency(f_i)**	**Mid value x_i**	$u_i=\frac{(x_i-A)}{h}$	$(f_i\times u_i)$
24.5 - 29.5	14	27	-3	-42
29.5 - 34.5	22	32	-2	-44
34.5- 39.5	16	37	-1	-16
39.5 - 44.5	6	42 = A	0	0
44.5 - 49.5	5	47	1	5
49.5 - 54.5	3	52	3	12
54.5 - 59.5	4	57	3	12
	$\Sigma f_i=70$			$\Sigma(f_i\times u_i)=-79$

$A=42, h=5, \Sigma f_i=70$ and $\Sigma(f_i\times u_i)=-79$

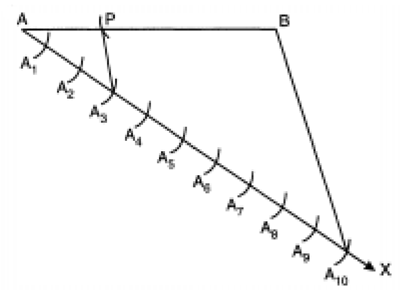
$\therefore \quad \text{mean}, \bar{x}=\left\{A+\frac{h\times(\Sigma f_i\times u_i)}{\Sigma f_i}\right\}$

$=42+\left\{5\times\frac{(-79)}{70}\right\}$

$=42-5.64$

$=36.36$

Section D

35. 

We have to draw a line segment of length 5 cm and divide it in the ratio 3 : 7.We write the steps of construction as follows:

Steps of Construction:

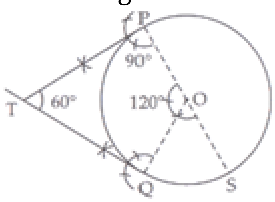
i. Draw a line segment AB = 5 cm.
ii. Draw any ray AX making an acute angle down ward with AB.
iii. Mark the poitns A_1 ,A_2 ,A_3 .... A_{10} on AX such that $AA_1 = A_1A_2 .... = A_9A_{10}$.
iv. Join BA_{10}.
v. Through the point A_3 draw a line parallel to BA_{10}. To meet AB at P.

   Hence AP : PB = 3 : 7

OR

Angle between tangents is 60°. So angles between their radii is 180° - 60° = 120°.

As the angles between tangents and their corresponding radii are supplementary.



Steps of construction:

i. Draw a circle of radius 4 cm.
ii. Draw any diameter POS.
iii. Draw OQ making ∠AOC = 120°.
iv. Draw tangent at P by drawing ∠OPT = 90°.
v. Similarly, draw ∠OQT equal to 90° to draw tangent.
vi. Both PT and QT tangents intersect at T and make angle of 60°.

Hence, the two tangents on circle are TP and TQ inclined at 60°.

Justification: Because the radius OP and tangent PT at contact point makes angle ∠TPO=90°

Similarly, ∠TQO = 90°

In quadrilateral TPOQ.

∠T + ∠P + ∠O + ∠Q = 360°

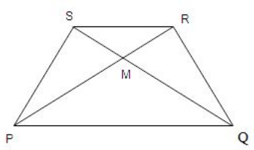
⇒ ∠T + 90° + 120° + 90° = 360° [∵ ∠O = 120° by construction]

⇒ ∠T = 360° - 300°

⇒ ∠T = 60°.

Hence, verified.

36. Given : $\Delta PMS \sim \Delta QMR$ and $PQ||SR$.



To show PS = QR

$\because \triangle PMS \sim \triangle QMR$

$\therefore \frac{PS}{QR} = \frac{PM}{QM} = \frac{MS}{MR}$ ...(i)

[corresponding sides of similar triangles are proportional]

Now, consider $\triangle PMQ$ and $\triangle RMS$

In these triangles, we have

$\angle PMQ = \angle RMS$ [vertically opposite angles]
$\angle MPQ = \angle MRS$ [alternate angles]
$\because \triangle PMQ \sim \triangle RMS$ [AA criteria]
$\therefore \frac{PM}{RM} = \frac{MQ}{MS}$
[corresponding sides of similar triangles are proportional]
$\Rightarrow \frac{PM}{QM} = \frac{MR}{MS}$ ...(ii)
From Eq (i) and Eq (ii), we get
$\Rightarrow \frac{MS}{MR} = \frac{MR}{MS}$
$\Rightarrow MS^2 = MR^2$
$\Rightarrow MS = MR$
From Eq(i) , we get
$\therefore \frac{PS}{QR} = \frac{MS}{MR}$
$\frac{PS}{QR} = 1$
$\Rightarrow PS = QR$ Hence proved.

37. Let the fraction be $\frac{x}{y}$
$\frac{x-2}{y+1} = \frac{1}{2}$
2x - 4 = y + 1
2x - y = 5..........(1)
Also, $\frac{x+4}{y-3} = \frac{3}{2}$
or, 2x + 8 = 3y - 9
or, 2 x - 3 y = -17........... (2)
Subtracting eqn. (1) from eqn. (2),
2y = 22
$\therefore$ y=11
Substituting this value of y in eqn. (i),
2x -11=5
2x = 5 + 11
2x = 16
$\therefore$ x=8
Hence, Fraction = $\frac{8}{11}$

OR

Suppose the speed of the train be x km/hr and the speed of the car be y km/hr.
CASE I
Distance covered by car is $(600-120)km$ = $480km.$
Now, Time taken to cover 480 km by train $\frac{120}{x}\text{hrs}\ [\because \text{Time} = \frac{\text{Distance}}{\text{Speed}}]$
Time taken to cover 480 km by car =$\frac{480}{y}$ hrs
$\therefore \frac{120}{x} + \frac{480}{y} = 8$
$\Rightarrow 8\left(\frac{15}{x} + \frac{60}{y}\right) = 8$
$\Rightarrow \frac{15}{x} + \frac{60}{y} = 1$
$\Rightarrow \frac{15}{x} + \frac{60}{y} - 1 = 0$ ....(i)
CASE II
Distance travelled by car is $(600-200)km$ = $400km$
Now, Time taken to cover $200km$ by train = $\frac{200}{x}$ hrs
Time taken to cover $400km$ by train = $\frac{400}{y}$ hrs
In this case the total time of journey is 8 hour 20 minutes
$\therefore \frac{200}{x} + \frac{400}{y} = 8 \text{ hrs } 20\ minutes$
$\Rightarrow \frac{200}{x} + \frac{400}{y} = 8\frac{1}{3}\ [\because 8 \text{ hrs } 20 \text{ minutes } = 8\frac{20}{60}\text{hrs} = 8\frac{1}{3}\text{hrs}]$

$\Rightarrow \frac{200}{x} + \frac{400}{y} = \frac{25}{3}$

$\Rightarrow \quad 25\left(\frac{8}{x} + \frac{16}{y}\right) = \frac{25}{3}$

$\Rightarrow \quad \frac{8}{x} + \frac{16}{y} = \frac{1}{3}$

$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} = 1$

$\Rightarrow \quad \frac{24}{x} + \frac{48}{y} - 1 = 0$ ....(ii)

Putting $\frac{1}{x} = u$and $\frac{1}{y} = v$in equations (i) and (ii), we get

15 u+ 60v -1=0 ....(iii)

24u+ 48v -1=0 ....(iv)

By using cross-multiplication, we have

$\frac{u}{60\times-1-48\times-1} = \frac{-v}{15\times-1-24\times-1} = \frac{1}{15\times48-24\times60}$

$\Rightarrow \frac{u}{-60+48} = \frac{-v}{-15+24} = \frac{1}{720-1440}$

$\Rightarrow \quad \frac{u}{-12} = \frac{v}{-9} = \frac{1}{-720}$

$\Rightarrow \quad u = \frac{-12}{-720} = \frac{1}{60}$ and $v = \frac{-9}{-720} = \frac{1}{80}$

Now, $u = \frac{1}{x} \Rightarrow \frac{1}{60} = \frac{1}{x} \Rightarrow x = 60$

and, $v = \frac{1}{y} \Rightarrow \frac{1}{80} = \frac{1}{y} \Rightarrow y = 80$

Speed of train = $60 km/hr$

Speed of car=$80 km/hr$.

38. Given

Volume of cistern = $150 \times 120 \times 110 \mathrm{cm}^3 = 1980000 \mathrm{cm}^3$

Volume of water = 129600 cm^3

Volume of one brick = $22.5 \times 7.5 \times 6.5 \mathrm{cm}^3 = 1096.875 \mathrm{cm}^3$

Each brick absorbs one - seventeenth of its volume of water

Volume of water absorbed by one brick = $\frac{1}{17} \times$ volume of brick

= $\frac{1}{17} \times 1096.875 \mathrm{cm}^3$

= 64.52 cm^3

Let n be the total number of bricks which can be put in the cistern without water overflowing. Then,

Volume of water absorbed by n bricks = $n \times \frac{1}{17} \times 1096.875 \mathrm{cm}^3$

$\therefore$ Volume of water left in cistern = $= \left(129600 - \frac{n}{17} \times 1096.875\right) \mathrm{cm}^3$

Since the cistern is filled upto the brim.

Therefore, Volume of the cistern = Volume of water left in the cistern + Volume of bricks

1980000 = $129600 - \frac{n}{17} \times 1096.875 + n \times 1096.875$

$n \times 1096.875 - \frac{n}{17} \times 1096.875 = 1980000 - 129600$

$1096.875 \times \left(n - \frac{n}{17}\right) = 1850400$

$1096.875 \times \frac{16n}{17} = 1850400$

$17550 \times \frac{n}{17} = 1850400 \Rightarrow n = \frac{1850400\times17}{17550} = 1792.41$

since the number of bricks cannot be in decimals

therefore, required number of bricks = 1792

OR

Diameter of the well = 2 m

Radius of the well, r = 1 m = 100 cm

Depth of the well, h = 14 m = 1400 cm

Height of embankment, H = 40 cm

Let the width of embankment = x cm

According to the question,

Volume of embankment = Volume of earth dug out

$\Rightarrow \quad \pi\left(R^2 - r^2\right) H = \pi r^2 h$

$\Rightarrow \quad \frac{22}{7}\left[(100+x)^2 - (100)^2\right] \times 40 = \frac{22}{7} \times 100 \times 100 \times 1400$

$\Rightarrow \quad (100+x)^2-(100)^2 = \frac{22\times100\times100\times1400}{7\times40} \times \frac{7}{22}$

$\Rightarrow$ 10000 + x^2 + 200x - 10000 = 350000

$\Rightarrow$ x^2 + 200x - 350000 = 0

$\Rightarrow$ x^2 + 700x - 500x - 350000 = 0

$\Rightarrow$ x(x + 700) - 500(x + 700) = 0

$\Rightarrow$ (x + 700)(x - 500) = 0

If x + 700 = 0

x = -700

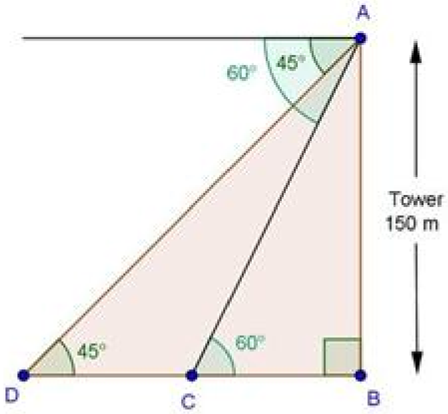
or,

If x - 500 = 0

x = 500

Since, the width cannot be negative.

Therefore, width of the embankment, x = 500 cm = 5 m

39. 

Let AB be the tower of height 150 m and two objects are located when top of tower are observed, makes an angle of depression from the top and bottom of tower are 45^0 and 60^0

In ΔABD

$\tan 45^0 = \frac{AB}{BD}$

$\Rightarrow 1 = \frac{150}{BD}$

$\Rightarrow BD = 150m$

In ΔABC

$\tan 60^o = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{150}{BC}$

$\Rightarrow BC = \frac{150}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\Rightarrow BC = \frac{150\sqrt{3}}{3}$

$\Rightarrow BC = 50\sqrt{3} = 50 \times 1.732 = 86.6m$

$\therefore$ Distance between two objects = DC

$= BD - BC$

= 150 - 86.6

= 63.4 m

40.

Class Interval	Mid - value	$d_i = x_i - 15$	$u_i = (x_i - 15)/6$	Frequency f_i	$f_i u_i$
0 – 6	3	-12	-2	-1	-14
6 – 12	9	-6	-1	5	-5
12 – 18	15	0	0	10	0
18 – 24	21	6	1	12	12
24 – 30	27	12	2	6	12
				N = 40	$\sum f_i u_i$ = 5

Let the assumed mean be (A) = 15
h=6
Mean = $A + h\frac{\sum f_i u_i}{N}$
= $15 + 6\left(\frac{5}{40}\right)$
$= 15 + 0.75$
$= 15.75$