

Solution
Class 10 - Mathematics
Sample Paper 1

Section A

1. (d)
a rational number or irrational number

Explanation:

The difference of two distinct irrational numbers can be either a rational number or an irrational number.

e.g difference between π and $(\pi - 3)$ is equal to 3 which is rational

$\sqrt{2}$ and $\sqrt{2} + 1$ both are irrational but their difference is 1 which is rational

similarly $\sqrt{2}$ and $\sqrt{3}$ are irrational and their difference $(\sqrt{3} - \sqrt{2})$ is also irrational

2. (b)
other than 2 or 5 only

Explanation:

A rational number can be expressed as a **non-terminating** repeating decimal if the denominator has the factors other than 2 or 5 only.

3. 4

4. The prime factors of 32875 are

$$32875 = 5 \times 5 \times 5 \times 263$$

clearly, this prime factorization of 32875 is unique.

5. (c)
 $\frac{7}{9}$

Explanation:

Let the fraction be $\frac{x}{y}$.

According to question

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$\Rightarrow 11x + 22 = 9y + 18$$

$$\Rightarrow 11x - 9y = -4 \dots\dots\dots(i)$$

$$\text{And } \frac{x+3}{y+3} = \frac{5}{6}$$

$$\Rightarrow 6x + 18 = 5y + 15$$

$$\Rightarrow 6x - 5y = -3 \dots\dots\dots(ii)$$

On solving eq. (i) and eq. (ii), we get

$$x = 7, y = 9$$

Therefore, the fraction is $\frac{7}{9}$

6. $2x^2 + px + 3 = 0$

$$D = p^2 - 4 \times 2 \times 3 = p^2 - 24$$

For real roots, $D \geq 0$

$$\Rightarrow p^2 - 24 \geq 0$$

$$\Rightarrow p^2 \geq 24$$

$$\Rightarrow p \geq \sqrt{24} \text{ or } p \leq -\sqrt{24}$$

$$\Rightarrow p \geq 2\sqrt{6} \text{ or } p \leq -2\sqrt{6}.$$

7. 5

8. For consecutive terms of AP,

$$2(2k^2 + 3k + 6) = (3k^2 + 4k + 4) + (4k + 8)$$

$$\Rightarrow 4k^2 + 6k + 12 = 3k^2 + 8k + 12$$

$$\Rightarrow k^2 - 2k = 0$$

$$\Rightarrow k(k - 2) = 0 \Rightarrow k = 0 \text{ or } k = 2$$

9. equilateral

10. In Δ 's ADC and BEC, we have

$$\angle ADC = \angle BEC = 90^\circ \text{ [Given]}$$

$$\angle ACD = \angle BCE \text{ [Common]}$$

So, by AA-criterion of similarity, we obtain

$$\Delta ADC \sim \Delta BEC$$

11. (a)
0

Explanation:

Since coordinates of any point on y -axis is $(0, y)$.

Therefore, abscissa is 0.

12. (a)
-5

Explanation:

Since x -coordinate of a point is called abscissa.

Therefore, abscissa is -5.

13. (b)
 $AP = \frac{1}{2} AB$

Explanation:

$$AP = \sqrt{(2 - 4)^2 + (1 - 2)^2}$$

$$= \sqrt{4 + 1} = \sqrt{5} = \text{units}$$

$$AB = \sqrt{(8 - 4)^2 + (4 - 2)^2}$$

$$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

Here $AB = 2 \times AP$

$$\therefore AP = \frac{1}{2} AB$$

14. (c)
-1

Explanation:

$$\text{Given: } \cot^2 \theta - \frac{1}{\sin^2 \theta}$$

$$= \frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta}$$

$$= \frac{\cos^2 \theta - 1}{\sin^2 \theta}$$

$$= \frac{-\sin^2 \theta}{\sin^2 \theta} = -1$$

$$[\because 1 - \cos^2 \theta = \sin^2 \theta]$$

15. (b)
9

Explanation:

$$9 \sec^2 A - 9 \tan^2 A$$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9(1) = 9$$

Hence, the correct choice is 9.

16. (b)

Explanation:

$$\text{Since } \sec \theta = \sqrt{1 + \tan^2 \theta}$$

$$\therefore \sec \theta = \sqrt{1 + (\sqrt{3})^2}$$

$$= \sqrt{1 + 3} = \sqrt{4} = 2$$

17. In Fig, OA is the radius and AC is the tangent from the external point C.

Therefore, $\angle OAC = 90^\circ$ (Theorem: Radius and tangent are always perpendicular to each other at the point of contact)

$$\angle BOC = \angle OAC + \angle ACO \text{ (Exterior angle property)}$$

$$\Rightarrow 130^\circ = 90^\circ + \angle ACO$$

$$\Rightarrow \angle ACO = 130^\circ - 90^\circ = 40^\circ$$

OR

$$QP = 3.8$$

QP = PT (Length of tangents from the same external point are equal)

Therefore, PT = 3.8 cm

Also, PR = PT = 3.8 cm

Now, QR = QP + PR

$$QR = 3.8 + 3.8 = 7.6 \text{ cm.}$$

18. 1350cu. cm

19. (b)
median

Explanation:

The measure of central tendency that can be obtained graphically is Median.

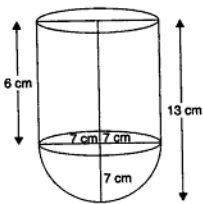
The median is less affected by outliers and skewed data distribution i.e. when the distribution is not symmetrical.

20. elementary event

Section B

21. \therefore Diameter of the hollow hemisphere = 14 cm

$$\therefore \text{Radius of the hollow hemisphere} = \frac{14}{2} = 7 \text{ cm}$$



\therefore Radius of the base of the hollow cylinder = 7 cm

Total height of the vessel = 13 cm

\therefore Height of the hollow cylinder = 13 - 7 = 6 cm

\therefore Inner surface area of the vessel = Inner surface area of the hollow hemisphere + Inner surface area of the hollow cylinder

$$= 2\pi(7)^2 + 2\pi(7)(6)$$

$$= 98\pi + 84\pi = 182\pi$$

$$= 182 \times \frac{22}{7} = 26 \times 22 = 572 \text{ m}^2$$

22. Total number of all possible outcomes = 52.

i. There are 13 cards of spades including one ace and there are 3 more aces.

So, the number of favourable cases = 13 + 3 = 16.

$$\therefore P(\text{getting a card of spades or an ace}) = \frac{16}{52} = \frac{4}{13}$$

ii. There are 2 black kings.

$$\therefore P(\text{getting a black king}) = \frac{2}{52} = \frac{1}{26}$$

iii. There are 4 jacks and 4 kings.

So, the number of cards which are neither jacks nor kings
 $= \{52 - (4 + 4)\} = 44.$

$\therefore P(\text{getting a card which is neither a jack nor a king})$
 $= \frac{44}{52} = \frac{11}{13}$

iv. There are 4 kings and 4 queens.

So, the number of cards which are either kings or queens
 $= 4 + 4 = 8.$

$\therefore P(\text{getting a card which is either a king or a queen})$
 $= \frac{8}{52} = \frac{2}{13}$

OR

Number of all possible outcomes is 36.

Let E be the event of getting the sum less than 7 on the two dice.

Then, the favourable outcomes are

$(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1).$

Number of favourable outcomes = 15.

$\therefore P(E) = \frac{15}{36} = \frac{5}{12}$

23. $\because PT = PQ$

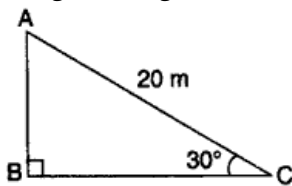
$\therefore PQ = 7 \text{ cm}$

Also $SR = QR$

$\therefore QR = 4$

Now, $RP = PQ - QR = 7 - 4 = 3 \text{ cm}$

24. In right triangle ABC,



$\sin 30^\circ = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{AB}{20} \Rightarrow AB = 10 \text{ m}$

Hence, the height of the pole is 10 m.

25. Since G is the mid-point of PQ, then,

$PG = GQ$. According to the question, $GH \parallel QR$. Therefore by basic theorem of proportionality, we have,

$$\frac{PG}{GQ} = \frac{PH}{HR}$$

$$1 = \frac{PH}{HR} \text{ [as } PG = GQ]$$

$$PH = HR$$

Hence, H is the mid-point of PR.

OR

According to the question, We have, $DE \parallel BC$, $DE = 4 \text{ cm}$, $BC = 8 \text{ cm}$ and $\text{area}(\triangle ADE) = 25 \text{ cm}^2$

In $\triangle ADE$ and $\triangle ABC$

$\angle A = \angle A$ [Common]

$\angle ADE = \angle ABC$ [Corresponding angles]

Then, $\triangle ADE \sim \triangle ABC$ [By AA similarity]

By area of similar triangle theorem

$$\frac{\text{Area}(\triangle ADE)}{\text{Area}(\triangle ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{25}{\text{Area}(\triangle ABC)} = \frac{4^2}{8^2}$$

$$\Rightarrow \text{Area}(\triangle ABC) = \frac{25 \times 64}{16} = 100 \text{ cm}^2$$

26. Let the first three terms in A.P. be $a - d$, a , $a + d$. It is given that the sum of these terms is 33.

$$\therefore a - d + a + a + d = 33$$

$$\Rightarrow 3a = 33$$

$$\Rightarrow a = 11$$

It is given that

$$a_1 \times a_3 = a_2 + 29$$

$$(a-d)(a+d) = a + 29$$

$$a^2 - d^2 = a + 29$$

$$121 - d^2 = 11 + 29$$

$$d^2 = 121 - 40 = 81$$

$$d = \pm 9$$

If $d = 9$ then the series is 2,11,20,29

If $d = -9$ then the series is 20,11,2,-7,-16

Section C

27. Since, the three persons start walking together.

\therefore The minimum distance each should walk so that all can cover the same distance in complete steps will be equal to LCM of 80,85 and 90.

Prime factors of 80,85 and are as following

$$80 = 16 \times 5 = 2^4 \times 5$$

$$85 = 5 \times 17$$

$$90 = 2 \times 9 \times 5 = 2 \times 3^2 \times 5$$

$$\text{So LCM of 80,85 and 90} = 2^4 \times 3^2 \times 5 \times 17$$

$$= 16 \times 9 \times 5 \times 17 = 12240$$

\therefore Each person should walk the minimum distance

$$= 12240 \text{ cm} = 122 \text{ meter } 40 \text{ cm}$$

OR

If possible, let $\sqrt{6}$ be rational and let its simplest form be $\frac{a}{b}$ then, a and b are integers having no common factor other than 1, and $b \neq 0$.

$$\text{Now, } \sqrt{6} = \frac{a}{b}$$

$$\Rightarrow 6 = \frac{a^2}{b^2} \text{ [on squaring both sides]}$$

$$\Rightarrow 6b^2 = a^2 \text{(i)}$$

$$\Rightarrow 6 \text{ divides } a^2 \text{ [}\therefore 6 \text{ divides } 6b^2 \text{]}$$

$$\Rightarrow 6 \text{ divides } a$$

Let $a = 6c$ for some integer c

putting $a = 6c$ in (i), we get

$$a^2 = 36c^2$$

$$6b^2 = 36c^2 \text{ [} 6b^2 = a^2 \text{]}$$

$$\Rightarrow b^2 = 6c^2$$

$$\Rightarrow 6 \text{ divides } b^2 \text{ [}\therefore 6 \text{ divides } 6c^2 \text{]}$$

$$\Rightarrow 6 \text{ divides } b \text{ [}\therefore 6 \text{ divides } b^2 = 6 \text{ divides } b \text{]}$$

Thus, 6 is a common factors of a and b

But, this contradicts the fact that a and b have no common factor other than 1

The contradiction arises by assuming that $\sqrt{6}$ is rational.

Hence $\sqrt{6}$ is irrational.

28. Given polynomial is $p(x) = x^3 - 3x^2 + x + 1$

Let α , β and γ are the zeros of polynomial

$$\text{So, } \alpha = (a - b), \beta = a \text{ and } \gamma = (a + b)$$

$$\text{Now, } \alpha + \beta + \gamma = -\frac{(-3)}{1}$$

$$\Rightarrow (a - b) + a + (a + b) = 3$$

$$\Rightarrow a + a + a - b + b = 3$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

$$\text{Also, } \alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{1}$$

$$\Rightarrow (a - b)a + a(a + b) + (a + b)(a - b) = 1$$

$$\Rightarrow a^2 - ab + a^2 + ab + a^2 - b^2 = 1$$

$$\Rightarrow 3a^2 - b^2 = 1$$

$$\Rightarrow 3(1)^2 - b^2 = 1 \quad [\because a = 1]$$

$$\Rightarrow 3 - b^2 = 1$$

$$\Rightarrow b^2 = 3 - 1$$

$$\Rightarrow b^2 = 2$$

$$\Rightarrow b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \pm\sqrt{2}$

$$29. a(x + y) + b(x - y) = a^2 - ab + b^2 \quad \text{..(i)}$$

$$a(x + y) - b(x - y) = a^2 + ab + b^2 \quad \text{..(ii)}$$

Rearrange (i) $\times (a - b)$ and (ii) $\times (a + b)$

$$(a - b)[(a + b)x + (a - b)y] = a^2 - ab + b^2$$

$$(a + b)[(a - b)x + (a + b)y] = a^2 + ab + b^2$$

$$(a^2 - b^2)x + (a - b)^2y = (a - b)(a^2 - ab + b^2) \quad \text{..(iii)}$$

$$(a^2 - b^2)x + (a + b)^2y = (a + b)(a^2 + ab + b^2) \quad \text{..(iv)}$$

On subtracting (iv) from (iii), we get

$$[(a - b)^2 - (a + b)^2]y = a^3 - a^2b + ab^2 - ba^2 + ab^2 - b^3 - b^3 - a^3 - a^2b - ab^2 - ba^2 - ab^2$$

$$-4aby = -4a^2b - 2b^3$$

$$-4aby = -2b[2a^2 + b^2]$$

$$y = \left[\frac{2a^2 + b^2}{2a} \right]$$

Put in eq. (i), we have

$$(a + b)x = a^2 - ab + b^2 - (a - b) \left(\frac{2a^2 + b^2}{2a} \right)$$

$$2a(a + b)x = 2a^3 - 2a^2b + 2ab^2 - 2a^3 - ab^2 + 2a^2b + b^3$$

$$2a(a + b)x = ab^2 + b^3 = b(a + b)$$

$$x = \frac{b^2}{2a}$$

OR

According to question the given system of equations are

$$3x + y = 1 \quad \text{.....(1)}$$

$$\text{and } kx + 2y = 5 \quad \text{.....(2)}$$

i. Since we know that,

The given equations are of the form

$$a_1x + b_1y + c_1 = 0 \quad \text{and}$$

$$a_2x + b_2y + c_2 = 0$$

has a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$$\text{Thus, } \frac{3}{k} \neq \frac{1}{2} \Rightarrow k \neq 6$$

Thus, k can take any real values except 6.

ii. We know that,

The given equations are of the form

$$a_1x + b_1y + c_1 = 0 \quad \text{and}$$

$$a_2x + b_2y + c_2 = 0$$

has no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\text{Therefore, } \frac{3}{k} = \frac{1}{2} \neq \frac{-1}{-5} \Rightarrow k = 6$$

$$30. a_n = 3 + 2n$$

Put $n = 1, 2, 3, \dots$

$$a_1 = 5, a_2 = 7, a_3 = 9, \dots$$

$$a = 5, d = 7 - 5 = 2$$

$$S_{24} = \frac{24}{2} [2 \times 5 + (24 - 1) \times 2] = 12[10 + 46] = 672$$

31. Let $A(11, -8)$ be the given point and let $P(x, 0)$ be the required point on x-axis

Now distance between point A and point P is given by , $AP = [(x-11)^2 + (0+8)^2]$ (1)

It is given that $PA = 10$ units (squaring on both sides)

$$\Rightarrow (PA)^2 = 100 \text{ [(substitute in ..(1)]}$$

$$\Rightarrow (x - 11)^2 + (0 + 8)^2 = 100$$

$$\Rightarrow x^2 + 121 - 22x + 64 = 100$$

$$\Rightarrow x^2 - 22x + 185 - 100 = 0$$

$$\Rightarrow x^2 - 22x + 85 = 0$$

$$\Rightarrow x^2 - 17x - 5x + 85 = 0$$

$$\Rightarrow x(x - 17) - 5(x - 17) = 0$$

$$\Rightarrow (x - 17)(x - 5) = 0$$

$$\Rightarrow x = 17 \text{ or } x = 5$$

Hence, the required points are (17, 0) and (5, 0).

32. Taking L.H.S. : $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta}$

$$= \frac{\sin \theta (1 - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}$$

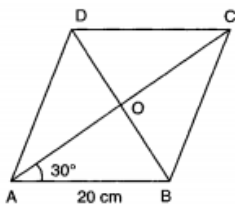
$$= \frac{\sin \theta (\sin^2 \theta + \cos^2 \theta - 2 \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - \sin^2 \theta - \cos^2 \theta)} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{\tan \theta (\cos^2 \theta - \sin^2 \theta)}{(\cos^2 \theta - \sin^2 \theta)}$$

$$= \tan \theta$$

Hence Proved, LHS = RHS

OR



According to question, we first draw a rhombus ABCD of side 20 cm and $\angle BAD = \angle BCD = 60^\circ$. Then we will find length of diagonals.

Note that the diagonals of a rhombus are perpendicular bisectors of each other and diagonals AC and BD are bisectors of $\angle BAD$ and $\angle ABC$ respectively.

So, $\triangle AOB$ is a right triangle such that,

$$\angle BAO = 30^\circ, \angle AOB = 90^\circ$$

and $AB = 20$ cm.

$$\therefore \cos \angle BAO = \frac{OA}{AB}$$

$$\Rightarrow \cos 30^\circ = \frac{OA}{20}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OA}{20}$$

$$\Rightarrow OA = \frac{\sqrt{3}}{2} \times 20 = 10\sqrt{3}$$

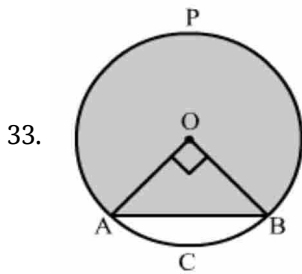
$$\text{Also we have, } \sin \angle BAO = \frac{OB}{AB}$$

$$\Rightarrow \sin 30^\circ = \frac{BO}{20}$$

$$\Rightarrow \frac{1}{2} = \frac{BO}{20}$$

$$\Rightarrow BO = \frac{20}{2} = 10$$

$$\therefore AC = 2 \times 10\sqrt{3} = 20\sqrt{3} \text{ cm, } BD = 2BO = 2 \times 10 = 20 \text{ cm}$$



According to the question, we are given that,

Radius of the circle = $r = 7$ cm

Central angle = $\theta = 90^\circ$

$$\text{Area of the minor segment} = \frac{\pi r^2 \theta}{360} - \frac{1}{2} r^2 \sin \theta$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{90}{360} - \frac{1}{2} \times 7 \times 7 \times \sin 90^\circ$$

$$= \frac{77}{2} - \frac{49}{2}$$

$$= \frac{77-49}{2}$$

$$= \frac{28}{2}$$

$$= 14 \text{ cm}^2$$

$$\text{Area of a circle} = \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

Area of the major segment = Area of a circle - Area of the minor segment

$$= 154 - 14 = 140 \text{ cm}^2$$

34. Let the assumed mean (A) = 2

No of children x_i	No of families f_i	$f_i x_i$
0	10	0
1	21	21
2	55	110
3	42	126
4	15	60
5	7	35
	$\sum f = 150$	$\sum f_i x_i = 352$

$$\text{Average number of children per family} = \frac{352}{150} = 2.35 \text{ (approx)}$$

Section D

35. Given, $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$

$$\frac{(x-3) + (x-1)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{x-3 + x-1}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2x-4}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2(x-2)}{(x-1)(x-2)(x-3)} = \frac{2}{3}$$

$$\frac{2}{(x-1)(x-3)} = \frac{2}{3}$$

$$(x-1)(x-3) = 3$$

$$x^2 - 4x + 3 = 3$$

$$x^2 - 4x = 0$$

$$x(x-4) = 0$$

$$x = 0, x - 4 = 0$$

$$x = 0, x = 4$$

OR

Let shorter side of rectangle = x metres
 Let diagonal of rectangle = $(x + 60)$ metres
 Let longer side of rectangle = $(x + 30)$ metres
 According to pythagoras theorem,

$$(x + 60)^2 = (x + 30)^2 + x^2$$

$$\Rightarrow x^2 + 3600 + 120x = x^2 + 900 + 60x + x^2$$

$$\Rightarrow x^2 - 60x - 2700 = 0$$

Comparing equation $x^2 - 60x - 2700 = 0$ with standard form $ax^2 + bx + c = 0$,
 We get $a = 1$, $b = -60$ and $c = -2700$

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{60 \pm \sqrt{(60)^2 - 4(1)(-2700)}}{2 \times 1}$$

$$\Rightarrow x = \frac{60 \pm \sqrt{3600 + 10800}}{2} \Rightarrow$$

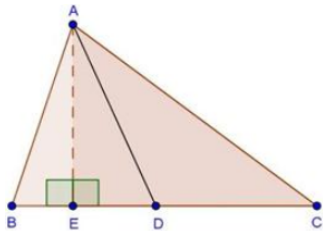
$$\Rightarrow x = \frac{60 \pm \sqrt{14400}}{2} = \frac{60 \pm 120}{2} \Rightarrow 4$$

$$x = \frac{60 + 120}{2}, \frac{60 - 120}{2}$$

$$\Rightarrow x = 90, -30$$

We ignore -30. Since length cannot be in negative.
 Therefore $x = 90$ which means length of shorter side = 90 metres
 And length of longer side = $x + 30 = 90 + 30 = 120$ metres
 Therefore, length of sides are 90 and 120 in metres.

36. We have,



In $\triangle ABC$, AD is a median.

Draw $AE \perp BC$

In $\triangle AED$, by Pythagoras theorem, we have,

$$AD^2 = AE^2 + DE^2$$

$$AE^2 = AD^2 - DE^2 \dots \dots \dots (1)$$

In $\triangle AEB$, by pythagoras theorem

$$AB^2 = AE^2 + BE^2$$

$$\Rightarrow AB^2 = AD^2 - DE^2 + (BD - DE)^2 \text{ [using (1)]}$$

$$\Rightarrow AB^2 = AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$

$$\Rightarrow AB^2 = AD^2 + BD^2 - 2BD \times DE$$

$$\Rightarrow AB^2 = AD^2 + \frac{BC^2}{4} - BC \times DE \dots (2) \text{ [BC = 2BD given]}$$

Again, In $\triangle AEC$, by pythagoras theorem,

$$AC^2 = AE^2 + EC^2$$

$$\Rightarrow AC^2 = AD^2 - DE^2 + (DE + CD)^2 \text{ [using (1)]}$$

$$\Rightarrow AC^2 = AD^2 - DE^2 + DE^2 + CD^2 + 2CD \times DE$$

$$\Rightarrow AC^2 = AD^2 + CD^2 + 2CD \times DE$$

$$\Rightarrow AC^2 = AD^2 + \frac{BC^2}{4} + BC \times DE \dots \dots \dots (3) \text{ [BC = 2CD given]}$$

Add equations (2) and (3), we get,

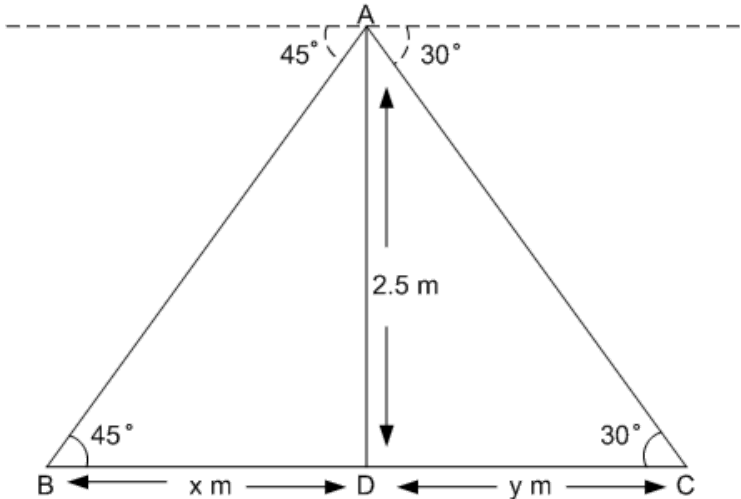
$$AB^2 + AC^2 = 2AD^2 + \frac{BC^2}{2}$$

$$\Rightarrow 2AB^2 + 2AC^2 = 4AD^2 + BC^2$$

$$\Rightarrow 4AD^2 = 2AB^2 + 2AC^2 - BC^2$$

$$\Rightarrow AD^2 = \frac{2AB^2 + 2AC^2 - BC^2}{4}$$

37. Given,



Let the width of the river be BC.

D is the point on the bridge and

$$AD = 2.5m$$

In right $\triangle ABD$,

$$\tan 45^\circ = \frac{AD}{BD}$$

$$\Rightarrow 1 = \frac{2.5}{x}$$

$$\Rightarrow x = 2.5m$$

In right $\triangle ADC$,

$$\tan 30^\circ = \frac{AD}{DC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2.5}{y}$$

$$\Rightarrow y = 2.5\sqrt{3}m$$

$$BC = x + y$$

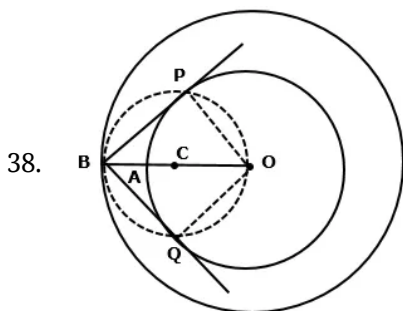
$$= 2.5 + 2.5\sqrt{3}$$

$$= 2.5(1 + \sqrt{3})$$

$$= 2.5(1 + 1.732)$$

$$= 6.83m$$

Therefore, width of the river is 6.83m.



38.

Steps of construction:

1. Take a point O on the plane of the paper and draw a circle of radius $OA = 4$ cm.
Also, draw a concentric circle of radius $OB = 6$ cm.
2. Find the midpoint C of OB and draw a circle of radius $OC = BC$.
Suppose this circle intersects the circle of radius 4 cm at P and Q.
3. Join BP and BQ to get the desired tangents from a point B on the circle of radius 6 cm.
By actual measurement, we find that $BP = BQ = 4.5$ cm

Verification:

In $\triangle BOQ$, we have $OB = 6$ cm and $OP = 4$ cm

$$\therefore OB^2 = BP^2 + OP^2$$

$$\Rightarrow BP = \sqrt{OB^2 - OP^2} = \sqrt{36 - 16} = \sqrt{20} = 4.47 \text{ cm} = 4.5 \text{ cm}$$

Similarly, BQ = 4.47 cm = 4.5 cm.

OR

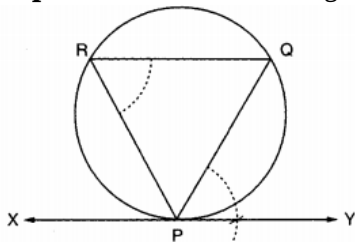
We follow the following steps of constructions:

STEP I Draw any chord PQ through the given point P on the circle.

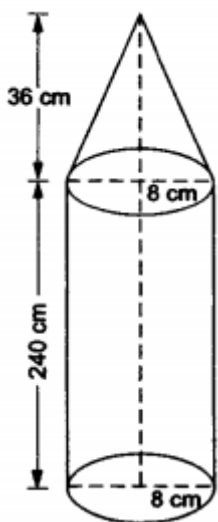
STEP II Take a point R on the circle and join P and Q to a point R.

STEP III Construct $\angle QPY = \angle PRQ$ and on the opposite side of the chord PQ.

Step IV Produce YP to X to get YPX as the required tangent.



39.



Let us suppose that r denotes the radius of the cylinder = 8 cm.

Suppose R denotes the radius of the cone = 8 cm.

Let h be the height of the cylinder = 240 cm.

Suppose H is the height of the cone = 36 cm.

Total volume of the iron = volume of the cylinder + volume of the cone

$$= \pi r^2 h + \frac{1}{3} \pi R^2 H = \pi r^2 \left(h + \frac{1}{3} H \right) \text{ [as } r=R=8\text{cm each]}$$

$$= \left[\frac{22}{7} \times 8 \times 8 \times \left(240 + \frac{1}{3} \times 36 \right) \right] \text{ cm}^3$$

$$= 50688 \text{ cm}^3$$

$$\therefore \text{Weight of the pillar} = \text{volume in cm}^3 \times \text{weight per cm}^3$$

$$= \left(\frac{50688 \times 10}{1000} \right) \text{ kg} = 506.88 \text{ kg}$$

Therefore, the weight of the pillar is 506.88 kg.

OR

Let us suppose that the Radius of the bigger end of the frustum (bucket) of cone = $R = 20$ cm

Suppose r denotes the Radius of the smaller end of the frustum (bucket) of the cone = 8 cm

Height = 16 cm

$$\text{Now, Volume} = \frac{1}{3} \pi h [R^2 + r^2 + Rr]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 16 [20^2 + 8^2 + 20(8)]$$

$$= \frac{352}{21} [400 + 64 + 160]$$

$$= \frac{352 \times 624}{21}$$

$$= \frac{219648}{21}$$

$$= 10459.43 \text{ cm}^3$$

Now,

$$\text{Slant height of the cone} = l = \sqrt{(R - r)^2 + h^2}$$

$$l = \sqrt{(20 - 8)^2 + 16^2}$$

$$l = \sqrt{12^2 + 16^2}$$

$$l = \sqrt{144 + 256}$$

$$l = \sqrt{400}$$

$$l = 20 \text{ cm}$$

Slant height is 20 cm

Now,

$$\text{Surface area} = \pi[R^2 + r^2 + (R + r) \times l]$$

$$= \frac{22}{7}[20^2 + 8^2 + (20 + 8) \times 16]$$

$$= \frac{22}{7}[400 + 64 + 448]$$

$$= \frac{22}{7} \times 912$$

$$= \frac{20064}{7}$$

$$= 2866.29 \text{ cm}^2$$

Thus, the surface area of the bucket is 28866.29

40. i. Less than series:

Weight(in kg)	Number of students
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

plot the points A(40,3), B(42, 5), C(44, 9), D (46,14), E(48,28), F(50,32) and G(52,35) on a graph paper.

Join all points freehand to get the curve.

ii. More Than Series:

Weight(in kg)	Number of students
More than 38	35
More than 40	32
More than 42	30
More than 44	26
More than 46	21
More than 48	7
More than 50	3

On the same graph paper as above, we plot the points P(38, 35), Q(40, 32), R(42,30), S(44, 26), T(46, 21), U(44, 7) and V(50,3).

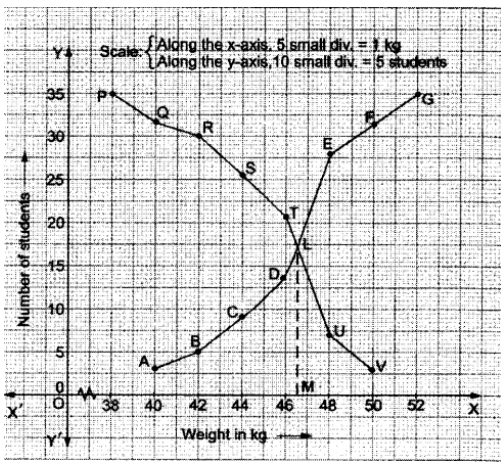
Join all points freehand to get the curve.

Graph:

Scale:

Along the x-axis, 5 small div. = 1 kg.

Along the y-axis, 10 small div. = 5 students.



The two curves intersect at the point L. Draw $LM \perp OX$.

$OM = 46.5 \text{ kg}$

\therefore median weight = 46.5 kg